# THE GROUND WORK
## OF
# DEDUCTIVE LOGIC

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# PREFACE TO THE SECOND EDITION

I very much regret that owing to some unavoidable reasons I could not have the Second Edition of my book published in due time. In preparing this edition I have thoroughly revised the First Edition and have made changes which, I hope, will make the book more suitable and useful to students of Intermediate Classes of Indian Universities. It will also provide a helpful handbook for those who may intend to pursue the study of the subject further.

In the present edition I have omitted some portions which I, in the light of my experience, considered difficult and unnecessary for Intermediate Students and have added some passages and sentences in order to make the topics easily intelligible to them. Portions printed in small types are chiefly meant for ambitious students and students of average merit may leave those portions out without any serious loss to themselves. In the first edition exercises were not provided in the first part of the book but in this edition they have been put at the end yith useful hints. These exercises will be helpful to the students.

I express my sincere indebtedness to those authors from whose writings I have drawn liberally and quoted freely in discussing different problems. I gratefully remember the help which was rendered to me by my teacher Prof. H. H. Crabtree, M.A. (Oxon), who is unfortunately no more, in the preparation of the first edition of my book. He prepared the table of contents, read all the proofs very carefully and prepared all the diagrams and notes thereof. As the second edition also amply bears the stamp of his labours I must once again acknowledge my indebtedness to him and must also express my thanks to the Calcutta University for allowing me to use the University questions in preparing the exercises contained in this book.

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# INTRODUCTION

## DEFINITION, NATURE AND SCOPE OF LOGIC

### Knowledge and its Sources

Logic is the *Science of Valid Thinking or Reasoning.* The object of Logic is to find out the conditions of acquiring true knowledge by avoiding error. By means of thinking we acquire knowledge. As the Science of Thinking or Logic has for its object the acquisition of valid knowledge or truth, we may at the outset consider the nature and function of knowledge and its sources.

Knowledge is progressive and since men often err it is necessary to investigate the conditions of valid thinking

Without thinking no knowledge is possible. Men as rational beings cannot do without thinking just as they cannot do without food. But the knowledge of one generation is often rejected by a subsequent generation. So it is true to say that the path of knowledge is strewn with the wreckage of exploded theories. Thus we come to find that most sciences give up old beliefs and accept new ones as a result of progressive investigation and enlightenment. The new astronomy of to-day is no longer the old astronomy of the Greeks, nor is modern physics the same as the crude physics of antiquity. So men have grown modest, and no longer think that knowledge is stereotyped, fixed and unchanging. They have discarded the belief that the accepted system of knowledge of one generation has a dictatorial function over all times to come. Though this is true, man has extended his mental activity to the conquest of the forces of Nature, and

is more and more mastering the secrets of the world. In spite of this, we cannot avoid errors altogether, and since failures are inevitable, it is necessary to investigate the conditions of right thinking.

But when does our thinking become right? When is our thought true, or in other words, what constitutes truth? Now, whenever we think, we think of some object, no matter if our thought is perceptual or conceptual. Thought begins with judgment. The statement 'This flower is red' has an objective reference, and is true if it agrees with the nature of the object to which it refers, that is, if the flower is really red. But this judgment or thought would be false if the flower were green or of some other colour, or if the object, to which the judgment refers, were something different from a flower. Similarly the judgment 'all material bodies gravitate' will be true if in the actual world material bodies without exception do gravitate. So we may say that our thought is true when it corresponds to or agrees with some aspect of reality, that is, with the actual world. We should here remember that truth and falsity can be predicated only of thought.

Our thought is true when it agrees with the actual world.

We may now distinguish between perceptual or immediate knowledge, and conceptual or mediate knowledge. When my knowledge of a certain object is direct, that is, when it is not acquired through the knowledge of some other object, it is immediate. Thus the knowledge of sensible objects, *e.g.* 'This man is tall,' 'This drink is sweet,' etc., is immediate and perceptual. Similarly, my knowledge of the fact that 'I am pleased,' or 'I desire something,' etc., is perceptual knowledge, and as such is immediate or direct. Some thinkers recognise also instances of immediate or intuitional knowledge that are not perceptual, *e.g.*, the knowledge that A is.

Immediate and mediate knowledge. Perception involves elements of thought.

A, that every event has a cause, etc. But even perceptual knowledge cannot strictly be regarded as purely immediate, that is, free from all elements of thought. Take the judgment 'This man is tall.' Though this judgment is perceptual, it has elements of thought in it, because without previous knowledge of man and of tallness I could not have formed the judgment. To recognise the object of my perception as a tall man requires previous knowledge. Inferential knowledge is regarded as mediate knowledge because such knowledge is acquired through the mediation of the knowledge of other objects, that is, indirectly. I cannot have the knowledge that 'all men are mortal' without having any previous experience of particular men dying. Similarly we cannot have the judgment that 'water boils at 212° Fahr.,' without having the experience of particular cases of water boiling at that temperature. Again when I pass on to the conclusion 'Socrates is mortal' from the premises 'All men are mortal' and 'Socrates is a man', my knowledge is mediate or indirect because it is acquired through the knowledge of the two given premises.

Authoritative knowledge being mediate, the main sources of knowledge are perception and inference.

Besides the two sources of knowledge, *viz.*, perception and inference, another source is often recognised, *viz.*, authority or verbal testimony. F t knowledge acquired through authority or verbal testimony may not properly be regarded as derived from a distinct source. It is perhaps inferential though it has some appearance of directness. A large part of our knowledge is acquired through authority, and all the knowledge of the past to which we have access has been preserved for us through language which is the instrument of thought. The knowledge that we acquire through the study of great scientists, historians and philosophers appears to be immediate and direct but really such knowledge is mediate. I accept as true the views of those authors and the testimony of those persons in whom I have confidence or who can stand criticism.

**What is Logic?**

What we have already discussed will now enable us to understand what Logic really is. Etymologically, the term 'Logic' means a study which is concerned with reason and language. The main purpose of Logic, however, is to teach us how to reason correctly; but since there is a close connection between reasoning and language, Logic is also concerned with the right use of language.

The main function of Logic is to enable us to reason correctly in order to attain true knowledge.

The object of reasoning or thinking is the attainment of truth. When we look at the sky and find that it is cloudy, we say that it will rain. This is reasoning. Again, when we say to a boy that if he gets into the river he will be drowned as he does not know how to swim, what we actually do is merely to reason with him or infer something about the situation. Reasoning consists in passing from something given to something not given. On the basis of our experience we say that, fire warms, water quenches thirst, all men make mistakes, etc., and we arrive at such propositions by means of reasoning. The attainment of truth is not possible without right reasoning. Knowledge is not only power, but also something without which we cannot succeed in practical life. So reasoning or thinking correctly has immense practical utility.

But very often we make use of the instrument of language in order to carry on our thought processes or reasoning. In thinking out a complex mathematical problem we have to take the help of symbols. Ideas often remain vague unless they are put into language. And the language which a thinking being uses should be precise, clear and correct, and must be free from ambiguity. So a logician has not merely to investigate the conditions of correct reasoning; but he must at the same time study

But to attain true knowledge, correct use of language is necessary as it is the instrument of thought.

language, which is the instrument of accurate thought. Logic is thus indirectly concerned with the right use of language.

**Definition of Logic**

Before examining some of the outstanding definitions of logic we may at the outset formulate and explain the definition of logic that appears to us to be satisfactory. Logic has rightly been defined as the science of the principles of valid thought or reasoning. Every science systematically studies some branch of the world as it appears to us. Scientific knowledge is different from popular knowledge, which applies itself to the study of particular phenomena without trying to find out the inter-relation between them. Scientific knowledge seeks to be definite and accurate. Further, every science studies laws or principles which hold good uniformly of a number of objects between which these laws serve as connecting links. And when we have discovered laws relating to the definite groups of phenomena which particular sciences study, we may discover in addition that the different groups themselves are connected with one another; that is, they are inter-related and not isolated. Scientific knowledge is very useful because by studying a few fundamental principles we can gain knowledge of a very large number of objects. The things of the world are innumerable and no man can know everyone of them, but the study of them is reduced by science to the study of some fundamental principles which it is possible for the human mind to know in a general way. Thus botany studies the laws according to which different plants behave, originate and grow, and reduces the plant-world to a system. It is not possible for us to study all plants individually, for they are numberless, but we can have knowledge of all of them if we know the principles or laws which govern the plant-kingdom. Similarly we can have general knowledge of the behaviour of the planets if we have access to the science of astronomy, which systematically studies the principles

Definition of Logic, as the science of valid thought. Scientific knowledge and popular knowledge.

according to which they move. Similarly geometry studies spatial relations, and physics and chemistry, the relations existing between physical objects and their properties, and so on. Just as every other science studies laws or principles relating its own subject-matter, so also does logic study the principles of its own subject-matter, *viz.*, thought. We have already seen that thought may be either perceptual or conceptual, and that logic studies both these aspects of thought. Now 'Thought' may mean either the process of thinking or the product of thinking. Logic studies both the process and the product of thinking. But logic is not quite interested in studying thought as it is, but rather in thought as it ought to be in order to be valid or true. We have already seen that thought is valid when it agrees with the real world. Men think correctly or incorrectly, and logic by examining thought finds out the conditions of valid thinking.

Logic, Ethics & Aesthetics, the three normative sciences. Their nature explained.

We may further explain the nature of logic, which is a normative science, by comparing it with the two sciences recognised as normative, *viz.*, ethics and aesthetics. The ideal of logic is truth, that of ethics goodness and that of aesthetics artistic beauty. Logic is the science of valid thought, ethics of right conduct and aesthetics of artistic taste. Logic is concerned with thought, ethics with volition or will and aesthetics with a kind of feeling. The ideal of logic, however, *viz.*, truth, is clearer and more definite than the ideal either of ethics, which is the good life, or of aesthetics, which is artistic beauty.

Normative and positive science.

We may here explain the distinction between normative and positive science. A normative science studies a group of facts with reference to some ideal and lays down rules for its attainment. A positive science, on the other hand, studies a group of facts without reference to any ideal. So it is said that a positive science deals with facts as they are and a normative

science deals with them as they ought to be, that is, from the standpoint of an ideal. Psychology, Botany. Biology, Astronomy, Physics, Chemistry, Mechanics, etc., are positive sciences. They state the laws of operation of their subject-matter. Psychology tells us how men think under different conditions and what the general laws of thinking are. Logic also like Psychology deals with thinking but it tells us what principles men ought to follow in their thinking, so that their thinking or reasoning may be valid. Logic is thus a normative science and is so far different from the positive sciences.

Logic is a science and not an art but it may be regarded as a practical science. Its aim is not to lay down precepts to make men reason well.

We may now consider whether logic besides being a science is also an art. The Port Royal logicians define logic as "the Art of Reasoning," while Whately defines it as "the science, as well as the art, of reasoning." Many other logicians hold that logic is an art. Without criticising the above definitions in detail, we may ask ourselves whether logic can properly be regarded as an art. According to Joseph art may mean "practical skill in doing a thing or theoretical knowledge of the way in which it is best done." In the first sense cooking, carpentry, etc., are arts, in the second sense navigation, music, surgery, etc., may be regarded as arts. But if we interpret art in the second sense, it requires the help of other sciences. Thus navigation cannot be successful without some knowledge of astronomy, mechanics, meteorology, physics, mathematics, etc. In this sense logic is regarded as an art because it is supposed that the knowledge of the principles of valid thought enables men to reason correctly, avoid fallacies and detect errors. In this sense politics and ethics were regarded by Aristotle as arts, because according to him a knowledge of the principles of the state would enable a man to act successfully for the good of the state. Similarly the knowledge of the principles of good conduct would make a man act rightly. An art, therefore, is a body of precepts for performing some work. Thus logic is divided into theoretical

logic and practical logic. We do not deny that logic is practical, for it disciplines the mind well and a disciplined mind can think well and act well. But logic should not on that account be regarded as an art. As the aim of ethics is to define the ideal of good conduct, so the aim of logic is to define the ideal of truth. Logic no more teaches men to reason well and to avoid fallacies than ethics teaches them to act rightly. Logicians often argue falsely and ordinary men unacquainted with logic often reason correctly. Men thought rightly or wrongly and acted rightly or wrongly before the sciences of logic and ethics were recognised. So Locke says, "God has not been so sparing to men, to make them barely two-legged creatures, and left it to Aristotle to make them rational." The proper function of light is to dispel darkness, and when darkness is removed, men can move about easily without falling. But it is not the function of light to enable men to walk without falling. Similarly though the knowledge of the principles of logic may enable men very often to avoid fallacies and to reason correctly, its aim is not to lay down precepts to make them reason correctly. It is, however, undeniable that logic is a practical science. Though Mill recognises the theoretical aspects of logic, he, like many others, regards it as a practical science. So he defines logic as "the science which treats of the operations of the human understanding in the pursuit of truth," or as "the science of the operations of the understanding which are subservient to the estimation of evidence."

Logic cannot be regarded as a formal science pure and simple.

Formal logicians like Kant, Mansel, Ward, Hamilton, and others regard logic as the science of the formal principles of thought. They make a distinction between form and matter and suppose that logic studies the form of thought only and not its matter or content. Let us see what they mean by form. Form means that which is the same in many individuals materially different. Thus coins though materially different may have the same form. Similarly different horses,

different men, etc., have the same general form, though individual horses and men differ materially. But, as Aristotle has pointed out long ago, forms exist in individuals and there cannot be any form without matter. Logic does not study the formal principles of thought alone, but the conditions of valid thinking. Thought is valid only when it corresponds to the nature of the real world. Logic no doubt studies forms of thought, but it studies the matter or content of thought as well. If we regard logic merely as the science of the formal principles of thought, it may be suggested that there may be form without matter. But since truth has always objective reference, logic, though it studies the forms of thought, cannot ignore the matter of thought.

Since truth has always objective reference, logic cannot ignore the matter of thought.

Before closing this topic we may point out that logic is defined as the science of the regulative principles of thought or reasoning, because it guides or regulates our thinking for the purpose of attaining truth which is the ideal of logic.

## Logic and Language.

We have seen that logic is the science of valid thought. Thought has always an objective reference, that is, whenever we think, we think of something. An individual mind can think of the material world, of other minds, and can reflect upon itself as well. It can also think of imaginary objects, such as, ghosts, fairies, etc. So the object of thought may be the states of one's own consciousness or something other than the states of one's own consciousness, whether it be some other mind or some material or imaginary object external to himself. In a word the object of thought is the whole universe, consisting of objects both real and imaginary. We can think of this world as a unity or can reflect upon some aspect of it. In search of truth we try by means of our thought or judgment to interpret this world rightly. But we often think wrongly, and therefore logic,

Nature of Logic and why it is related to language.

by criticising thought, tries to systematise the principles of valid thinking. Thought which is true must be universal and necessary, that is, true to all men at all times; what is true in this sense can never be otherwise. Thus logic is concerned with universal thought and not with individual whims and fantasies. It studies the thoughts of normally constituted minds. Just as the universe is one, so is there unity in the different processes of thought. As logic studies universal thought, it requires the help of a universal instrument to express it. This universal instrument of thought is language.

We have previously remarked that there is close connection between thought and language and as such, logic is related to language. We may now consider how logic and language are related to each other. Expressive gestures, such as asking a person by the movement of the hand to come near, are the elementary forms of language. But logic is not concerned with such a language or instrument of thought. It is related to conventional language, that is, to written and spoken language, because by means of such language we can discriminate between different ideas. Language is as universal as thought itself. Language aids thought in various ways. Further we cannot carry on a complex process of thought without the aid of language or symbols, as in working out a difficult mathematical problem. But even though logic requires the help of language, we must not regard it as the science of language. We may now show in what ways language is an aid to logic.

The different ways in which language helps thinking

1. It is by means of language that we can analyse a complex notion into its constituent parts. Suppose a man is climbing a tree. The idea of a-man-climbing-a-tree is a complex idea, but the employment of language enables us to analyse it into the ideas of man, the act of climbing and the idea of a tree. Similarly, substance, attribute, cause and effect, etc., which are thought together, can be analysed or separated by means of language which helps the analysis of thought.

2. With the help of language, the formation of concepts, notions or general ideas becomes possible. Let us see how we form concepts. The concept 'horse' is the general idea of the class horse, and similarly the concept 'man' is the general idea of mankind. Now we form the concept 'horse' by first observing a number of horses and then comparing them with one another so as to find out their common characteristics or attributes. When these common characteristics are discovered, they are abstracted away, *i.e.*, isolated in thought from the peculiarities of each horse observed. After abstraction we generalise these attributes which are common to all individuals, and form the concept or notion 'horse.' But this notion or concept has to be named so as to retain it in memory and to give fixity of meaning to the concept, which otherwise would remain vague and indistinct. In this way the formation of concepts and their retention in memory becomes possible by means of language.

3. The process of thinking is simplified or shortened by means of language. When we have a number of names representing notions or general ideas, we can apply them to particular cases. Besides, it is difficult to form concepts of complex matters, *e.g.*, the British Constitution, the League of Nations, circulation of blood etc. In such cases names standing for them and describing them are very useful.

4. Language is a means of communicating thought. Men could not have expressed their thoughts to one another if there had been no language. It is by means of language that the ideas of one person are communicated to others, and such communication enriches the store of knowledge.

5. Language further preserves thought. We could not have access to what Plato or Aristotle or Copernicus or Newton thought if there were no language to preserve their ideas. We are rich in knowledge to-day because of this heritage, and the importance of language for enshrining the wisdom of those who preceded us cannot be over-estimated. Though it is true that language as

an instrument of thought is related to logic, "formal logic," as Keynes says, "is still concerned primarily with thought, and only secondarily with language as the instrument of thought."

Logic as the science of valid thinking must avoid ambiguity in the use of language. Words often have different shades of meaning, and ambiguity in words leads to confusion of thought and is a fruitful source of error. Words often change their meaning by generalisation, *e.g.*, oil, which originally meant olive oil, now means any kind of oil. Similarly by specialisation, words change their meaning, *e.g.*, fowl, which meant any bird, now means a particular domesticated species only. Accurate thinking depends upon the accurate use of words.

Logic must avoid the ambiguous use of language which leads to invalid thinking.

## Views regarding Conception, Judgment and Inference

We have already explained what a concept or notion is. But the problem is whether the human mind is really capable of forming concepts or general ideas. Three views have been expressed on this point. They are (1) Realism (2) Nominalism (3) Conceptualism.

The three recognised views regarding Conception, *viz.* Realism, Nominalism, and Conceptualism, explained.

(1) *Realism*: Plato is an advocate of realism. According to him the human mind is capable of forming concepts, and corresponding to these general notions there are real essences existing in the world of reason, which, though an ideal world, is regarded by Plato as more real than the known phenomenal world of ours. Thus corresponding to the notion 'virtue' which is in the mind, there is the essence or form or 'idea' of virtue in the invisible or intelligible world. Particular instances of virtue are merely copies or shadows of that essence. Similarly corresponding to the notion or concept 'man', which is in the mind, there exists the essence of man in the real world and individual men are mere copies or imperfect representations of that real and ideal essence. Aristotle however rejected this realism of Plato and pointed out that forms, 'ideas' or essences exist only in the individuals that

possess them, and not apart from them. (2) *Nominalism*: According to nominalists only names are general. It is not possible for the human mind to form general notions. Whenever we think, we think of some particular thing and not of any general idea. Corresponding to a general name there may be an individual mental image or a series of such images which are concrete and not abstract. Hobbes and Berkeley are among the chief advocates of this theory. Mill, though a nominalist, holds a modified view regarding the mind's capacity to form concepts. He holds that general names fix our attention upon some common attributes of a class indicated by the name, and that in the case of intense attention these attributes may exclude all others. (3) *Conceptualism*: According to conceptualists (Mansel, Sigwart, Kant, Ward, Stout, and others) concepts are not sensible images but are intelligible, that is, they are mental syntheses of general attributes. Thought can be carried on by means of general ideas or concepts without the help of any image, generic or other, that is, without reference to individual objects. Thinking does not necessarily imply pictorial thinking, that is, thinking by means of images. General names are but symbols of general ideas, concepts or notions. It appears to us that the conceptualists are right, since we do have such notions as virtue, justice, man, etc., which are not always accompanied by images but which we are justified in regarding as concepts by the fact that they are intelligible, that is, their meaning can be understood.

**Three views regarding judgment and inference.**

As with regard to conception, so in regard to judgment and inference logicians hold different views. (1) According to conceptualists (Hamilton, Mansel, and others) a judgment expresses a relation between two concepts, notions or ideas, whether the relation is one of inclusion or participation. Inference according to them consists in passing from one or more judgments to a conclusion. These formal logicians

The three views, viz. Conceptualistic. Nominalistic and Realistic or Objectivistic, regarding judgment and inference explained.

define logic as the science of the pure or formal laws of thought. They mean that the matter of thought is not important for logic. According to them truth means not correspondence but consistency of thought and its freedom from self-contradiction. An idea is true, according to them, if it does not involve self-contradiction and is consistent with the totality of our thought and experience. We shall explain this view of truth more fully hereafter. (2) According to nominalists a judgment is nothing but a statement about language. Every proposition simply expresses a relation between two terms or names, and inference or argument consists in passing from one or more propositions to a new one, that is, from one set of relations between them. Hobbes is the chief exponent of this view. Logic according to them is the science of names and their relation. (3) There is also a third school of logicians who are objectivists. According to them judgments express a relation between substances or between attributes or between substances and attributes. Take the proposition 'Man is mortal.' According to conceptualists the judgment means that there is a relation of agreement between the concept man and the concept mortal; according to nominalists it means that there is a relation of agreement between the name or term man and the name or term mortal, while according to objectivists or realists the judgment means that there is a relation between the things or attributes signified by the term man and the things or attributes signified by the term mortal. According to objectivists inference consists in passing from one or more sets of relations between substances and attributes to a new set of relations between them. Spencer is a pronounced objectivist. He defines logic as "the science which formulates the most general laws of correlation among existences considered as objective."

It may be pointed out that every judgment is an act of thought, and since it claims truth, it must have objective reference. Logic therefore is neither purely subjective nor purely objective. Further, since thought cannot be carried on in most

cases without the help of language, logic cannot altogether ignore consideration of language and must treat of terms, propositions, arguments, etc. Logic is the science of thought, and since thought is related both to objects and to language, it takes an interest, though a secondary one, in things and names.

**Formal and Material Logic**

The standpoint of formal logic.

A distinction is commonly drawn between formal logic and material logic as well as between formal truth and material truth. According to the formalistic view of logic, logic is concerned with formal consistency or formal truth, that is, it can ignore the matter of thought and study the forms of thought alone. We have already pointed out that even physics, chemistry, geometry, etc., are also in a sense formal sciences, since they require abstraction and deal with the forms of things. But since logic is more abstract than any physical science, it is supposed to be more formal than any other science. Bosanquet rightly remarks that a physicist or a chemist has his laboratory and he can often appeal to sense-experiences, while a logician has no such laboratory in which to experiment upon thought. According to formal logicians, when a judgment is free from self-contradiction it is formally true, and the formal truth of a judgment depends upon its being consistent with other judgments which are known to be true. Thus consistency of thought and freedom from self-contradiction are according to formal logicians the test of formal truth. Thus 'No A is A,' since it involves self-contradiction, is false, while 'A is A' is a true judgment, because it is free from self-contradiction and is consistent with the totality of thought. Such logicians try even to reduce inductive inference to formal treatment. We have already pointed out that the truth of a judgment depends as much upon consistency of thought as upon its correspondence with some aspect of reality.

Its view of truth one-sided. Purely formal treatment of logic not possible.

Logic, therefore, does not allow of purely formal treatment. The doctrine of definition, of division, of classification, of categories, of

predicables, etc., cannot be understood without some knowledge of reality or the universe. Formal logic is identified with deductive logic.

The nature of material logic. The distinction between formal or deductive logic and material or inductive logic is not absolute but relative. Logic however is primarily concerned with forms of thought.

By material logic is generally meant inductive logic. In deduction or deductive inference we pass from general propositions to a conclusion which is usually less general, and here consistency is the main test of truth; *e.g.*, All M is P, all S is M, therefore all S is P; All material bodies are extended, this stone is a material body, therefore this stone is extended. In induction or inductive inference on the other hand we pass from the facts of experience which are particular, to a conclusion which is general; *e.g.*, John is mortal, James is mortal, Joseph is mortal, etc.; therefore, all men are mortal. Thus in induction we cannot omit to consider the matter of thought or the facts of the world, and our argument being based upon observation of particular facts, the laws that govern induction are said to be laws of material logic as opposed to those of formal logic. We may however point out that even inductive logic can be treated formally up to a certain point, as we shall see when we discuss the problems of induction. Thus just as formal logic cannot ignore the consideration of the matter of thought, since every judgment refers to some content of reality, so also induction cannot ignore the form of thought altogether. Therefore the distinction between formal and material logic is relative and not absolute. We may further remark that deduction and induction are not two complementary processes but are two aspects of the same process of thought. We shall find that inductive generalisations require the help of deduction for their verification and their extension to unobserved regions. Though it is true that the distinction between formal and material logic is not absolute, yet deductive logic can be treated of to a very large extent formally. We should also remember that though logic cannot

ignore the matter of thought altogether, it is primarily or mainly concerned with the forms of thought.

### The Utility of Logic

We have already shown that logic is not an art, yet its practical value is undeniable. Its function is not to lay down rules for the guidance of every particular science and teach it how to draw conclusions, nor is it concerned with helping men to argue correctly in every particular case of reasoning. The art of reasoning, that is, how to argue correctly in a particular case, does not fall within the scope of logic, just as casuistry, or the art of deciding how one ought to act in every particular instance, does not fall within the scope of ethics. In spite of all the above remarks logic is a very useful study. It not only adds to our knowledge but the abstract nature of the study of logic disciplines our mind to a high degree. Logical study is an excellent propaedeutic or introduction to the study of other sciences. Though an acute mind can detect fallacies in particular cases of reasoning, logic alone can find out why a particular piece of argument is fallacious by comparing it with the principles of valid thought. Further a systematic criticism of thought enables us to find out the valid principles of thought. When a particular science draws a false conclusion from premises, it is logic which can find out the reason why the conclusion is untenable. Further it is also largely true that right thinking leads to right action. Hamilton says that in the world there is nothing great but mind and in mind there is nothing great but reason and a well balanced mind is the supremest possession of man. Logical study balances the human mind to a high degree.

### Procedure and Mode of Treatment

We shall treat the principles of deduction first as is usually done, since they are clear and simple and the knowledge of them is necessary for an understanding of the principles of induction.

Judgment or proposition being the unit of thought, the treatment of logic really begins with the discussion of judgment. Proposition being judgment expressed in language, we shall deal with its forms. But before doing that we shall treat of terms, which are the constituents of propositions, and in connection with them we shall treat of categories, predicables, the doctrines of definition and division, etc. Then after treating propositions we shall pass to the discussion of deductive inference, which may be either immediate or mediate. Mediate inference may be either syllogistic or inductive. In this volume we shall discuss the principles of deduction only, leaving the discussion of the principles of induction for another volume. We should also remember that different pieces of inference are interrelated and move towards a definite goal. At the end of the second volume therefore we shall treat of method, which is the orderly arrangement in discourse of trains of inferences. Further since thought and language are closely related we shall indifferently use the vocabularies both of conceptualists and of nominalists and shall speak of concepts, judgments, inferences, as well as of terms, propositions and arguments, in the same sense. We shall also use the terminology of objectivists or realists from time to time. Deductive fallacies will be discussed at the end of the first volume and inductive fallacies at the end of the second volume.

## LOGIC AND OTHER SCIENCES

General relation of logic to other sciences.

The scope of logic extends, as Johnson rightly remarks, into the domain of philosophy on the one hand, and that of the sciences on the other. So no rigid distinction can be drawn between the provinces of logic and metaphysics on the one hand and the provinces of logic and the special sciences on the other. Logic, it appears, is as akin to metaphysics as to the special sciences. But the spheres of logic and metaphysics should be distinguished as far as possible. Since every science

must conform to the regulative principles of thought which are the study of logic, logic in a sense influences every science and therefore is rightly called the science of sciences. Though this is true, we must not suppose that the influence which logic exerts on other sciences is more than general. It does not examine the evidence or data of the different sciences but only enquires whether valid conclusions are drawn from the evidence. Since logic is the science of proof, it has the right to weigh evidence in order to determine whether conclusions have been rightly drawn from it. To find out causal laws it is necessary for every science to draw conclusions according to logical principles. Logic is called Methodology because it criticises and examines the methods employed by different sciences. But on this account every science should not be regarded as a branch of logic. So it has rightly been said that logic is practical inasmuch as it forms an excellent propaedeutic or introduction to the study of other sciences. It has therefore been regarded by some as not merely the science of sciences but also as the art of arts. But we must not forget that logic is a practical science only in a secondary sense, for its function is not to lay down rules and precepts for the guidance of other sciences. It will be clear now that logic is not only an end in itself, but provides general guidance for the other sciences because it criticises all thought. Though logic is related to the sciences in general, it is supposed to be specially related to the science of mind (psychology), the science of being (metaphysics) and the science of language (grammar).

## Relation of Logic to Psychology

"Psychology treats of psychical states and processes, their objects as such and the conditions of their occurrence." (Stout). Psychical or mental states may be either states of thinking or states of feeling or states of willing, such as sensation, perception, recollection, fearing, hating, feeling pleasure or pain, longing, desiring, choosing, resolving, attending, etc. Logic is related to psychology inasmuch as they both deal with thinking, but while

psychology deals with the processes of thinking, logic deals both with processes and products of thinking. Psychology regards laws of reasoning as uniformities, while logic deals with them as regulative and authoritative laws of thought, determining the formal relations in which the products of thought stand to one another. Therefore while psychology is a positive and concrete science, logic is a normative and abstract science. In other words, psychology deals with the actual processes of thinking while logic deals with the ideal of reasoning and the processes to be followed for its realisation. Psychology tells us how we come to believe certain things, and how one idea gives rise to another idea according to the laws of association. Logic tells us how we ought to think and believe and how our ideas should be regulated so that they may be valid. Psychology deals with the origin and development of judgment and conception, but logic deals with them as they ought to be, that is, with the conditions which they must fulfil in order to be valid. Since logic criticises the method of psychology, as of every other science, it may be regarded as superior to psychology. But in a sense psychology is wider than logic, because it deals with all mental phenomena, namely, those of thinking, feeling and willing, while logic deals with the processes of judging, perceiving, conceiving and reasoning, that is, with all the processes of thought and its products. There cannot be any sound logic without some knowledge of psychology. Psychology of cognition therefore may be regarded as the basis of logic. Without knowing the actual processes of thinking we cannot determine how men ought to think. Though this is true, logic is not a branch of psychology. The spheres of logic and psychology of cognition very often overlap, but logical and psychological problems should be separated as far as possible. We have already remarked that like every other science psychology is dependent upon logic, because the latter gives required guidance to science.

**Logic and Metaphysics**

Metaphysics or Ontology investigates the nature of Being or

ultimate reality. It tries to answer the question whether the things that we experience are real or only phenomenal, whether there is an ultimate reality behind the phenomenal world or the world as it appears to us. Thus metaphysics does not accept anything as real without subjecting it to searching criticism. It begins with doubt and may or may not reach a positive result; that is, after examination of all that we know it may either say that knowledge of reality is possible or that it is not. Whatever may be the conclusions of different metaphysicians, they are all at one in criticising the presuppositions of the sciences, including those of logic. The sciences make such assumptions as that matter exists, that there is conservation of energy, that the things of the world are causally related, that nature is uniform, that knowledge of the laws of nature is possible, and so on. Metaphysics however cannot accept any of these assumptions as true without examinations. It therefore criticises the presuppositions of every science and goes beyond it. Logic may be regarded as intermediate between other sciences and metaphysics. Logic criticises the conclusions of the sciences and provides criteria of validity to which every science has to conform. But it generally accepts their assumptions. It accepts certain laws of thought without criticism as axiomatic, it supposes that knowledge in the absolute sense is possible, that the object of knowledge is real, that nature is uniform, that every event has a cause, that laws of Identity, Contradiction and Excluded Middle are axiomatic truth and so on. But Metaphysics critically examines all these assumptions of logic and thus goes beyond logic. Since logic enquires into the truth and falsity of thought and since thought is true only when it corresponds with some aspect of reality, logic cannot but assume that knowledge of reality is possible. But metaphysics makes no such assumptions. Its central problem is to determine whether we can know reality or not, whether knowledge is only relative or whether absolute knowledge is possible. It enquires whether all knowledge is the knowledge of appearance or whether there may be knowledge of reality as well. Metaphysics therefore criticises the

presuppositions of natural science, and in the same way it examines the presuppositions of logic. Thus though logic goes beyond all other sciences, metaphysics goes beyond logic as well. But logic cannot avoid entering into the domain of metaphysics to some extent. Logic tries to know this and that, but to have knowledge of them it requires some knowledge of the nature of the 'this' and of the 'that'. Logic however does not attempt to define the nature of reality or the ultimate being, if there be any such reality. It serves as a propaedeutic to the study of metaphysics and without some knowledge of logic the study of metaphysics is not possible. Further a metaphysician has also to reason validly to arrive at correct conclusions, and therefore metaphysics, like other sciences, is indebted to logic which studies the conditions of valid thinking.

**Logic and Grammar**

Since language is the instrument of thought, logic is supposed to be related to rhetoric and grammar, which are the sciences of language. Logic however cannot properly be regarded as specially related to rhetoric, which is concerned with the emotive use of language. Rhetoric is concerned with language in so far as it is intended to appeal to emotion, but logic has no reference to emotion. It is the science of reasoning and tries to find out the principles to which thought must conform in order to be true. Further logic cannot be supposed to be specially related to particular grammars. There are different grammars for different languages, but the science of logic is one, and always the same. Logic however may be regarded as related to universal grammar, if there is any such thing. Mr. Johnson holds that universal grammar should be subsumed under logic because the modes in which words are combined cannot be isolated from modes of thought, and also because negation, conjunction, disjunction, implication, and alternation are modes of logic as well as of grammar. Further he says that the grammatical analysis of sentences is analogous to the logical analysis of thought. Hence he holds that

grammar should be subordinated to logic. He does not however note the points of difference between logic and grammar. Logic is primarily concerned with the forms of thought and secondarily with language, but grammar is primarily concerned with the forms of language and secondarily with thought. Logic has nothing to do with non-significant words. Only significant words can form terms in a proposition. Logic takes no note of that division of words into parts of speech which is so marked a feature in the grammatical analysis of language. It is concerned only with those words which can be either the subject or the predicate of a proposition. Thus prepositions and conjunctions in their normal use can never be terms of a proposition. Grammar has long exercised a tyrannical influence over logic, but logic should be emancipated from its control as completely as possible.

## THE FUNDAMENTAL LAWS OF THOUGHT

"The laws of thought, regulative principles of thought or postulates of knowledge are those fundamental, necessary, formal and a priori mental laws in agreement with which all valid thought must be carried on." Three such laws are recognised by traditional logic, *viz.* (1) the Law of Identity, (2) the Law of Contradiction or of Non-contradiction, (3) the Law of Excluded Middle. Leibnitz's Principle of Sufficient Reason is not regarded as an a priori principle of thought by logicians, for reasons which will be explained later. Besides the three fundamental laws of thought, certain other postulates of knowledge are recognised but are not held to be so important as the three main laws. These fundamental laws of thought are regarded as a priori because they are not derived from experience, and yet they are assumed in all processes of reason exercised upon the facts of the world. They are formal because they provide the main types or patterns of thought which are most general and universal and are involved in all thinking: and also because they are not concerned with particular facts of experience and cannot by themselves ascertain the properties of them. They

are necessary because we cannot think of them as reversed, nor can we knowingly violate them. We commit fallacies no doubt, but we do so out of ignorance, not knowingly. The three fundamental laws are postulates of knowledge because without them knowledge cannot be systematised. They resemble scientific uniformities or laws, but differ from the latter inasmuch as they are not derived from particular experiences and are self-evident, that is, their truth is not to be proved by other judgments. But though these laws are self-evident, they do not provide any criterion or standard of how men ought to think. They only describe thought as it is. Keynes rightly remarks that the function of these laws is negative rather than positive. They give us principles by conforming to which we may avoid fallacies, but they do not lead to any positive result. Hamilton and others are wrong in trying to deduce all processes of valid thinking from the three laws of thought. We may now explain these laws.

**The Law of Identity**

The simplest statement of the principle of identity is 'A is A', Leibnitz states it as 'Everything is what it is' and Jevons as 'Whatever is, is'. The conception of identity is meaningless if it does not imply diversity as well. 'A is A,' to be significant, must imply diversity. In this case there is diversity because the two letters 'A' occupy different points in space and yet are thought of as the same. Thus when we say that a thing is identical with itself we mean that it remains the same amidst diversity of circumstances. We do not mean by the principle of identity that two things are the same without any attendant difference, which is nonsense; we only mean by it 'identity of one thing amid diversity in other things'. Sigwart says that the law implies that "Truth is something fixed and invariable". Bradley points out that the law of identity signifies that truth is at all times true; once true always true, once false always false. Thus the principle of identity tells us that if a judgment or term is repeated several times its truth is not affected. Mill

as a nominalist holds that the principle means that whatever is true in one form of words is true in every other form of words which conveys the same meaning, and that the same term should be used in the same sense throughout a discourse. Thus in place of the proposition "All men are mortal" we can, according to Mill, legitimately use the proposition, "All human beings are subject to death." In cases of other propositions we may do the same.

### The Principle of Contradiction

The principle of contradiction or of non-contradiction may be best stated as 'A cannot both be B and not B'. Jevons states it as 'Nothing can both be and not be.' It is also stated as 'A cannot be not-A,' or as 'A cannot be both B and not-B.' The principle means that the two contradictory judgments 'A is B' and 'A is not B' cannot both be true. The same subject cannot accept two incompatible attributes, or two contradictory terms cannot both be true of the same subject at the same time, *e.g.*, a thing cannot be green and not green. The truth of this principle is not affected if different parts of the same surface are green and not green, because here the attributes are referred to different points in space. The same portion, say, of the wall of a house cannot both be green and not green. Again the truth of the principle is not affected if the subject accepts incompatible or contradictory attributes at different points of time. Thus a piece of iron may be hot at one time and not hot at another time. But since here reference is to different points of time, we have really two propositions, *viz.*, that at such and such a time a piece of iron is hot and that at some other time it is cold. The principle is applicable when incompatible attributes are predicated of the same subject at the same time, that is, two contradictory propositions cannot both be true at the same time. Thus the two propositions 'A is B' and 'A is not-B' cannot both be true at the same time. The principles of identity and of contradiction form the basis of all

immediate inference and of some cases of mediate inference. Mill's interpretation of the Law of Contradiction is that "the affirmation of any assertion and the denial of its contradictory are logical equivalents which it is allowable and indispensable to make use of as mutually convertible." Thus to affirm 'A is B' is the same thing as to deny 'A is not-B'.

### The Law of Excluded Middle

This principle may be well stated as 'A either is, or is not B'. It has also been stated as 'A is either B or not-B.' The principle means that one of two contradictory propositions must be true. The principle of contradiction points out that two contradictory propositions cannot both be true, or two contradictory attributes cannot be accepted by the same subject. The principle of excluded middle supplements it by stating that one of the two contradictory propositions must be true or one of the two contradictory attributes must be accepted by the same subject. Thus a piece of iron must either be hot or not be hot: a man must either be honest or not be honest. One of the two propositions 'A is B' and 'A is not B' must be true. We need not like Sigwart deduce the principle of excluded middle from the principle of contradiction and that of double negation taken together. His principle of negation is that if we deny a negative proposition we get its contradictory affirmative proposition. Thus if the truth of 'A is not B' is denied or negated, then the truth of 'A is B' is to be affirmed. Jevons tries to combine the principle of contradiction and the principle of excluded middle by his 'principle of duality'. If two alternatives are exclusive and exhaustive, then they cannot both be true of the same subject at the same time, and one of them must be true of the subject. But it is better to keep the two laws independent without combining them into one. Just as in the principle of contradiction the reference to the same subject and to the same point of time is necessary, so also in the principle of excluded middle the reference must be to the same

subject and to the same point of time. The truth of the principle of excluded middle is denied by some. But these persons confuse contradiction with contrariety. It is pointed out that a flower need not be either white or black but may be red. A thing need not be either greater or less than another thing but may be equal to it. Similarly Mill says that between true and false propositions there may be unmeaning proposition. But in the examples given, the alternatives predicated of the same subject are not contradictories, but contraries. Thus it is undeniable that a flower must either be green or not be green, a thing must either be greater or not be greater than another thing. Green and red as well as greater and less are contrary terms but green and not-green as well as greater and not-greater are contradictory terms. In reply to Mill we may point out that an unmeaning proposition is no proposition at all. Thus the principle of excluded middle is as true as the other principles. Mill states the principle as, "It is allowable to substitute for the denial of either of two contradictory propositions the assertion of the other." We shall find that some instances of both immediate and mediate inference depend upon this principle.

The meaning of the three laws explained by comparison. These laws show that there is a close connection between logic and metaphysics because they are as much laws of thought as of things.

Mr. Joseph regards these laws of thought as ontological laws. So he says that, "The connection between questions about our thinking, and whatever we must think things to be, is excellently shown in the so-called laws of thought." These laws show that logic and metaphysics are closely related. According to Kant, change presupposes permanence, and this is the essence of the law of identity. The law of contradiction says that a thing cannot have opposite characters. The law of excluded middle states that it must have one of these characters. Since there cannot be any determination without negation, the principle of excluded middle is also necessary.

Joseph sums up the implication of the three laws in the following words:—"If we think about anything then (1) we must think that it is what it is; (2) we cannot think that it at once has a character and has it not; (3) we must think that it either has it or has it not." From the standpoint of language the three principles may be summed up in the following words of Welton and Monahan: "Whenever we use a term we must be understood to use it unambiguously both (1) positively and (2) negatively, and (3) it must either be given or denied to everything whatever. That is, the use of a term asserts all the attributes it implies, and denies all others which are incompatible with them; and everything must either possess all those attributes or be without some, or all, of them."

All these three laws are equally fundamental. We should not suppose that any of them is more important than the others. Some logicians regard the law of identity as more fundamental than the other two laws, while others suppose that the principle of contradiction is more important than the other two. But all three are equally necessary for our knowledge of the facts of the world. The three laws together imply that, "I affirm what I affirm, and deny what I deny; if I make any affirmation, I thereby deny its contradictory; if I make any denial, I thereby affirm its contradictory." (Keynes).

The three laws are equally important and necessary for knowledge.

## The Principle of Sufficient Reason

Leibnitz holds that the Principle of Sufficient Reason is as fundamental as the principles of Identity, of Contradiction and of Excluded Middle. He states the principle thus: "Whatever exists or is true must have a sufficient reason why the thing or proposition should be as it is and not otherwise." The principle may be symbolically expressed as A+B=C; *e.g.*, gunpowder and fire=explosion. It is a postulate of thought and implies that

things in the world are causally related. The law may mean either every event has a cause or that there must be some logical ground to explain why a proposition is true. It may be regarded as the foundation of syllogistic as well as of inductive reasoning. Since Leibnitz regards the principle of sufficient reason as implying mainly that the things of the world are causally related, it is therefore not sufficiently formal and cannot be regarded as a fundamental law of thought like the three principles of Aristotle already explained.

Some other postulates of knowledge recognised.

We shall discuss in its proper place Aristotle's '*dictum de omni et nullo*, which is regarded by him to be the postulate of syllogistic reasoning. The principle of causation as modified by the principle of uniformity of nature, which is supposed to be the formal ground of induction, will be treated of in connection with inductive inference. Hamilton gives us what he calls the Postulate of Logic. He says: "Before dealing with a judgment or reasoning expressed in language, the import of its terms should be fully understood, in other words, logic postulates to be allowed to state explicitly in language all that is implicitly contained in the thought." Since Hamilton allows the same meaning to be expressed by different words, his postulate is almost the same as Mill's reading of the Law of Identity. There may be principles other than the fundamental laws of thought upon which valid reasoning may depend. There are some such mathematical principles, *e.g.*, the argumentum a fortiori. 'If A is greater than B and B is greater than C then A is greater than C'; 'things equal to the same thing are equal to one another.' But we need not enumerate any other postulates of knowledge enunciated by logicians.

# CHAPTER I

## TERMS

### Introductory Remarks

Knowledge has its object.

Knowledge really begins with judging and every *judgment* has reference to reality, that is, it is concerned with matter of fact. All our judgments are based upon experience, and experience is experience of reality. Therefore it can be said that knowledge necessarily has reference to reality. Mere subjective attitude cannot be knowledge.

Judgment, Proposition and Sentence.

It is generally held that a proposition is the verbal expression of a judgment, but it would be better to regard proposition as judgment expressed in language because essentially judgment and proposition are the same. A proposition is a sentence but every sentence is not a proposition. 'Man is mortal' is a proposition and it is at the same time a sentence; but 'come here', 'will you come', 'I wish you well', 'may you live long,' etc. are sentences but not propositions. A sentence that is a proposition must be either true or false. The relation between proposition and sentence will be more fully discussed in the chapter on proposition.

The parts of a proposition: Subject, Predicate and Copula.

An analysis of a proposition gives us *three factors viz. Subject, Predicate, and Copula.* Something of which an affirmation or denial is made is called the subject, something that is affirmed or denied of the subject is called the predicate; while a mood of verb 'to be' (whether alone or accompanied by 'not'), by which the assertion is made, is called the copula. Let us take the proposition 'Socrates is wise.' Here 'Socrates' is

the subject, 'wise' is the predicate and 'is' is the copula. Again in the proposition, 'Man is not a four footed animal,' 'man' is the subject, 'a four-footed animal' the predicate and 'is not' the copula. We shall hereafter find that the copula is not a third term in a proposition. Every fully developed proposition has only *two terms,* viz., the *subject* term and the *predicate* term.

Terms and Names.

Some logicians use 'term' and 'names' in the same sense. A *name* is a symbol of some substance or attribute and represents some idea. Such words as Ram, Man, White, Virtue are names and they may be used as terms. A name in order to be a logical term must either be the subject or the predicate of a proposition. Therefore, all names are not terms though they may function as terms, that is, isolated names which are not parts of any proposition are not terms.

Term, idea and concept.

Every term represents some substance or attribute real or imaginary. Further, every term represents some idea. A term may stand either for an individual idea as in the case of the term 'India'; or, it may stand for a general idea or concept as in the case of the term 'man' or 'virtue'. Thus we find that a term has both an objective and a subjective aspect.

Single-worded and many-worded terms.

A term may consist of a single word or of a combination of words. "The Prime Minister of India is an Indian" is a proposition in which the subject term, 'the Prime Minister of India' and the predicate term, 'an Indian' are both many-worded terms. But the terms 'happiness' and 'desired' in the proposition 'happiness is desired' are both single-worded terms.

Categorematic and syncategorematic words.

A *categorematic* word is one which by itself can serve as a term, while a *syncategorematic* word is one which cannot itself function as a term but can become a part of a term only when joined with a categorematic word. Every word must be either categore-

matic or syncategorematic. It is plain that the nominative cases of nouns and pronouns are categorematic words, and so also are adjectives and participles, while the objective cases of nouns and pronouns are syncategorematic words. Adverbs, articles, prepositions, conjunctions and interjections are syncategorematic words because normally they cannot by themselves serve as terms. Adjectives can be used as predicates, but not as subjects except by an ellipsis, as in 'the poor are miserable.' In the proposition 'The is an article,' the term 'the' has been used as the subject of a proposition but in this case it has been used as a noun and not as an article.

Classification of terms

Terms are classified by logicians as—

1. Singular and General:
2. Collective and Distributive;
3. Concrete and Abstract:
4. Positive and Negative (Privative);
5. Absolute and Relative;
6. Particular and Universal;
7. Univocal, Equivocal and Analogous.
8. Connotative and Non-connotative.

In this chapter we shall discuss the first seven divisions. We shall devote a separate chapter to the last division, *viz.*, connotative and non-connotative terms. Of these divisions the first, third, sixth and eighth are important for logic. Though each division is exhausive, the distinctions between different divisions are not exclusive of one another. Though a term cannot be both singular and general, it can be general, concrete and connotative at the same time.

## Singular and General Terms

Terms may be either *Singular* or *General.* According to Keynes, "A singular or individual name is a name which is understood in the particular circumstances in which it is

employed to denote some one determinate unit only," while "a general name is a name which is actually or potentially predicable in the same sense of each of an indefinite number of units."

Definition of Singular and General terms.

In other words, a singular term stands for a single object while a general term denotes any one of a number of objects belonging to a class.

A singular term which refers to one object only may be either a *proper name* or a *designation,* that is, a uniquely descriptive name, as Welton calls it. A proper name is a sign or a mark which is predicated of an individual thing without signifying attributes which may be possessed by that thing. It is a mark of identification only. The term 'London' is the name of a place and it does not, from the logical point of view, signify any attribute which the place may possess, such as its situation, its importance, etc. A proper name may suggest certain qualities by association, *e.g.*, the name 'John' may suggest the appearance of the person John to one who knows him. But that is a psychological fact which should be distinguished from the logical implication of a name. 'John' as a proper name is a mark to identify a person and does not refer to any quality which that person may have.

Singular terms either proper names or uniquely descriptive names. Nature of a proper name.

It is no doubt true that different places or persons or things may have the same name, but the same proper name is not given to a number of objects because they possess some common attribute. In each use of the same name 'Jane,' whether it refers to a woman or a dog or a ship, it has a unique reference and points to one object only.

The nature of a proper name is not impaired even if many objects bear the same name.

*Significant singular names,* also called *designations* or *uniquely descriptive* names, should be distinguished from proper

A significant singular name refers to one thing only indirectly through its meaning. The characteristics of uniquely descriptive names.

names. A uniquely descriptive name refers to one object only, not, like a proper name, directly, but indirectly through its meaning. Thus 'the present President of the Indian National Congress' refers to one man only, because he possesses certain attributes. Here the reference is to an object through its meaning. Significant singular names are given to things because it is not always convenient to give proper names to all objects.

Nature of a General term.

We have already pointed out that a *general name* is applicable in the same sense to each of a number of similar objects, whether such objects be actual or potential. Such names are also called *Class Names,* since the reason for giving a general name to a number of objects in the same sense is that they have certain characteristics in common.

General names and uniquely descriptive names, similar yet different.

A general term resembles a uniquely descriptive term inasmuch as they both refer to objects indirectly through their meaning. But whereas a general term can be affirmed in the same sense of a number of objects, a designation or a significant singular term can be affirmed in the same sense of one thing only.

A general name may be transformed into a significant singular name by means of an individualising prefix.

A general name can be transformed into a uniquely descriptive name by means of some individualising prefix. 'Man' is a general term but 'this man' is a singular term. Here the addition of the individualising prefix 'this' has transformed the general term 'man' into a uniquely descriptive term. Similarly the term 'king' is general but the term 'the present King of England' is a singular term.

According to Welton such terms as 'Conqueror of England' or 'Emperor of Switzerland' are general names. The term

A name may be general if it refers to possible or imaginary objects having a common meaning.

'Conqueror of England' is general, though applicable to William I only, because it is potentially applicable to others, that is, it is conceivable that there might be other conquerors of England as well. The term 'Emperor of Switzerland' is also general, though Switzerland had no emperor, because it is conceivable that a number of persons might or may be emperors of Switzerland. Such terms as 'ghosts', 'fairy', etc., are general names.

The nature of unity of a general and of an individual name.

The unity of a class-term consists in its meaning while that of a singular term consists in its application. The term 'man' has one meaning though it is applicable to a number of objects but the term 'Ram' applies to one object and so its unity is to be found in its application.

### Collective and Distributive Terms

Collective name defined.

Keynes defines a *collective* name as "one which is applied to a group of similar things regarded as constituting a single whole," *e.g.*, army, navy, the Himalayas, the alphabet, library, the House of Commons, etc. A collective name may be either singular or general.

Collective terms may be either proper names or significant singular names or general names.

There are collective names which are similar to proper names, such as the Himalayas, the Alps, the Pyrenees, etc. There are other collective terms which are of the same nature as uniquely descriptive names or significant singular names, *e.g.*, the British navy, the Sanskrit alphabet, Hindu society, the Calcutta University Library, etc. 'The 40th Regiment of foot' is a singular collective name, but the term 'regiment' is a general collective name, because there are numerous regiments and the term 'regiment'

can be applied to each of them. Similarly, Library, University, School, Government, Jury, etc., are general collective names.

Though we cannot assert incompatibility between a collective and a general term, yet there is a real distinction between the *collective* and *distributive* use of names. "When we use a term collectively our assertion will only apply to the group as a whole; when we use it distributively we assert something about each member of the group individually." (Welton and Monahan). When we say that 'the British government has decided to enter into an economic agreement with the United States' we use the term 'British government' collectively. But when we say that 'the British government are trying to settle a dispute between the employers and the employed' we refer to the members constituting the government distributively, and here the term 'the British government' has its distributive use. The term 'man' when distributively used applies to every man, but when collectively used it applies to mankind. Again when we say that 'all the books on the shelf weigh 10 seers,' we use the term 'all the books' collectively, but we use the term distributively when we say that 'all the books on the shelf are text-books of logic.' Again, 'all the angles of a triangle are equal to two right angles' is a proposition exemplifying the collective use of the term 'all the angles,' while the same term is used distributively in the proposition 'all the angles of a triangle are less than two right angles.'

The distinction between collective and distributive use of names explained by examples.

**Concrete and Abstract Terms**

A concrete term is the name of an object, while an abstract term is the name of an attribute considered by itself. If this definition is accepted then all adjectives, which are names of attributes, become concrete, since 'white' means 'white things', 'red' 'red things', 'sweet' 'sweet things', and so on. But when an attribute is considered by itself without reference to

Explanation of the distinction between concrete and abstract.

something of which it is the attribute, the name of such an attribute may be called an abstract name, *e.g.*, whiteness, virtue, squareness, triangularity, etc. Attributes always belong to objects but they may be thought of either in relation to them or by themselves. In the former case they are concrete, in the latter they are abstract. From the metaphysical point of view, substance is that which has attributes, while an attribute is that which belongs to some substance.

It is possible for men to think of attributes as residing in some subject or to think of them without reference to any subject. The latter process of thought is called abstraction. When we say, 'Milk is white', we think of the attribute 'white' as a characteristic of milk, and therefore the term 'white' is concrete. But when we say 'whiteness is a pleasant colour,' we think of whiteness by itself and therefore 'whiteness' is abstract.

Concrete and abstract terms generally go in pairs; *e.g.*, beautiful—beauty, powerful—power,. white—whiteness, strong—strength, man—humanity, triangle—triangularity, father— paternity, etc. In all these cases the first term of each pair is concrete, while the second is abstract.

**Discussion as to whether abstract names are singular or general.**

Concrete names may be either general, *e.g.* man, house, etc., or singular, *e.g.*, John, James, etc. But there is a good deal of controversy as to whether abstract names are singular or general. According to Mill such abstract terms as admit of variety are general; e.g., colour, because there may be different colours; whiteness, since there are different shades of whiteness; virtue, because there are different kinds of virtue; while those abstract terms which do not admit of variety are singular, e.g., milk-whiteness, visibility, squareness, equality, etc. Some logicians have regarded all abstract names as singular, because according to them every abstract term indicates a single attribute. But those logicians are really in the wrong who hold that all abstract terms are general, on the ground that every

attribute may be possessed by a number of objects; for we give abstract names to attributes not because they belong to objects but because they are thought of by themselves. It is undeniable that when concretely used abstract names may be general; *e.g.*, in the propositions 'some virtues are more praised than others,' 'all yellows are pleasant looking', the terms 'virtue' and 'yellow' are used in a general sense.

**Positive and Negative Terms**

According to Keynes, "A *positive* name implies the presence, in the things called by the name, of a certain special attribute or set of attributes, while a *negative* name implies the absence of one or other of certain specified attributes."

**Definition of positive and negative terms.**

The distinction between positive and negative terms may be brought out by contrasting pairs of contradictory terms. Thus if A is positive, not-A is negative: 'white' is positive, 'not-white' negative: 'good' positive, 'not-good' negative: 'man' positive, 'not-man' negative.

**Examples.**

It would be profitable here to analyse the import of negative terms in greater detail.

What is the meaning of a negative term, say, not-A? According to Mill, if a positive term is significant, the corresponding negative term must also be significant, because the non-possession of an attribute is itself an atttribute. Thus if A is significant, not-A is also significant.

In a sense, again a negative term is indefinite and infinite, because not-A does not signify any attribute in particular and may signify an infinite number of attributes not signified by A; *e.g.*, if A implies 'red', then not-A implies not only white, blue, but also virtue, square, triangle, man, dog, etc. But it would be better not to regard a negative term as either indefinite or

**In a sense, negative terms are indefinite and infinite.**

infinite. 'Not-white' means any colour other than white and also refers to a limited universe of discourse, *viz.*, colour. 'Not-white' does not mean virtue, justice, square, and everything else which does not possess the attribute 'white'. It means a colour which is other than white. So Bradley says that every negation has a positive background: such a positive meaning however is not explicit in a negation though it is referred to by it. Keynes also holds that since negation always implies a particular universe of discourse, it cannot be meaningless, because it has a definite denotation, that is, can be interpreted as indicating a definite number of objects. Thus 'not-white' denotes or indicates all the colours other than white. We cannot therefore agree with Welton that a negative term implies the mere absence of some attribute and not the presence of any attribute whatever.

Some terms negative in appearance but positive in meaning.

Language has devised terms which have the appearance of negative terms but which cannot properly be regarded as negative. Such distinctions as happy—unhappy, pleasant—unpleasant, convenient—inconvenient, holy—unholy, sincere—insincere, sensible—senseless, fortune—misfortune, etc., are necessary from the practical point of view. None of these pairs, however, is an example of contradictory terms, because they are not exhaustive. Between 'pleasant' and 'unpleasant' there may be an indifferent state of feeling. Again 'unhappy' does not merely mean absence of happiness but it also means the presence of pain. Similarly 'unpleasant' has a positive meaning. So these terms may be regarded as positive, though negative in appearance.

Contradictory and contrary terms.

We may now consider the distinction between contradictory and contrary terms. Both contradictory and contrary terms are incompatible, that is, they cannot be true of the same subject at the same time. 'A, not-A; happy, not-happy; red, not-red;' etc., are examples of contradictory terms. But white-green, right-

left, happy-unhappy, greater-less, male-female, wise-foolish, etc., are examples of contrary terms. Contradictory terms are mutually exclusive and collectively exhaustive. Such contradictory terms as red and not-red exclude each other and they together exhaust the whole universe of colour. A thing must be either red or not-red; it cannot be both. Contrary terms like contradictory terms are mutually exclusive. But they are not collectively exhaustive. A thing cannot be both green and yellow at the same time, but it may be neither green nor yellow, but red. Thus we find that two contrary terms cannot both be true of the same subject at the same time, yet they may both be false of the same subject. A man may not be either happy or unhappy but may be indifferent. In the case of contradictory terms no intermediate position is possible. Thus between white and not-white there cannot be any intermediate colour. But between two contrary terms an intermediate position is possible as they are not together exhaustive. Between white and black there may be several intermediate colours.

Privative terms.

A *privative* term implies the absence of an attribute in a subject normally capable of possessing it. A privative name, according to Mill, is equivalent in its signification to a positive and a negative name taken together. Such terms as blind, deaf, dumb, lame, are privative terms. We call a person deaf because he lacks the capacity of hearing which we expect a human being to possess under normal condition. We do not call a stone and a tree deaf, because they are not expected to have the capacity of hearing. A man who is devoid of sight is called blind, and this term is privative because he might conceivably have had sight. A privative term means two things, *viz.*, the absence of a certain attribute and the presence of others from which the presence of the former might naturally be expected. Such terms as unhappy, ignorant, cruel, unkind, may also be regarded as privative. We call a man unkind, since, though kindness is a quality he might possess, he lacks it when he is unkind. To sum up, a positive

term implies the presence of something, while a negative term implies its absence, and a privative term implies the absence of an attribute in an object in which it might be expected to be present.

## Relative and Absolute Terms

Definition of relative and absolute terms, with examples.

Terms are further classified as Relative and Absolute or non-relative. Mill prefers to avoid the term 'absolute', since it has a good deal of metaphysical significance and uncertainty. Such pairs of terms as father, son, like, unlike, equal, unequal, ruler, subject, husband, wife, partner, friend, etc., are *relative* terms, while man, dog, house, tree, etc., are *absolute* terms. According to Keynes, "A name is said to be relative, when, over and above the object that it denotes, it implies in its signification another object, to which in explaining its meaning reference must be made." So it may be said that relative terms go in pairs. 'Father' implies 'son', 'husband', 'wife'; similarly 'like' implies 'like', 'equal', 'equal', 'unequal', 'unequal' 'friend', 'friend', and so on. Mill says, "Every relative name which is predicated of an object, supposes another object (or objects), of which we may predicate either that same name or another relative name which is said to be the *correlative* of the former." Thus 'son' is the correlative of 'father' and 'father' of 'son'. Each of the two related terms is called the correlative of the other.

Correlative terms have a common ground.

There is a common ground or basis upon which correlative terms rest. According to Mill, "All that appears necessary to account for the existence of relative names, is, that whenever there is a fact in which two individuals are concerned, an attribute grounded on that fact may be ascribed to either of these individuals." This attribute or ground of relation of two correlative terms is called by the Schoolmen the *funda-*

*mentum relationis.* The ground of relation between two partners is the fact of partnership, between two friends friendship, between husband and wife the fact of the marriage tie. Thus, according to Welton, "Paternity and sonship are not two different facts but the same fact viewed from two different sides and connoted both by parent and by son". Though 'friend' and 'like' are relative terms, 'friendship' and 'likeness' are not, since they are the grounds of relation. According to Mill a relation such as paternity, likeness, friendship is an attribute like any other attribute, say, whiteness, goodness, redness, etc. "To predicate of A that he is the father of B, and of B that he is the son of A, is to assert one and the same fact in different words. Father connotes the fact, regarded as constituting an attribute of A; son connotes the same fact, as constituting an attribute of B."

Relation according to Mill is an attribute.

Relative terms are general, because every relative term admits of a variety of instances. Thus many persons may be regarded as fathers, or wives, or friends, or sons or masters.

Relative terms general.

Jevons argues that since water is related to its constituent elements, gas to coal, a tree to the soil, every term is in a sense relative. We cannot understand the meaning of a term except by distinguishing it from another term. The psychological law of relativity also tells us that we can understand a particular state of mind only when we relate it to other mental states. Moreover, every term implies its contradictory—A implies not-A, white not-white. All notions and all things are thus relative. Nevertheless Jevons admits that when we call a term relative, we mean a special kind of relation which it bears to another term. Keynes, in replying to Jevons, holds that when we regard a name as relative we do not mean that the object it signifies cannot be thought of without some other object, or that it depends upon other objects, but we mean that the

Examination of Jevons' view that all terms are relative.

signification of a relative term cannot be understood without reference to something else which is called by a correlative name, as in the case of husband and wife.

### Univocal, Equivocal and Analogous Terms

A term may be used *univocally* or *equivocally*. Univocal and equivocal terms are not two distinct classes of terms, but they represent two different modes of using terms. According to Mill, "A name is univocal or applied univocally with respect to all things of which it can be predicated in the same sense; it is equivocal or applied equivocally as respects those things of which it is predicated in different senses." Thus man, house, tree are univocal terms, while file, light, foot are equivocal terms. 'File' may mean either a steel instrument or a line of soldiers. 'Foot' may mean either infantry or a unit of measurement. Owing to equivocal use of terms many fallacies are committed. In logic we must always avoid equivocation and must use a term in the same sense throughout a discourse.

An *analogous* term is predicated of two things not in the same sense but in similar signification, *e.g.*, brilliant light, brilliant man; a high mountain, a high sound; the foot of a man and the foot of a mountain. In the first pair of examples the term 'brilliant' is used almost equivocally. The point of similarity is that in the case both of light and of man the term 'brilliant' signifies some excellence. When we speak of a foot in reference to a man and to a mountain, analogy is to be sought in the fact that 'foot' implies that on which something stands. Owing to the metaphorical use of language we often reason falsely.

Simple and composite terms.

The distinction between *simple* and *composite* terms is nothing more than the distinction between single-worded and many-worded terms. A single-worded term is simple, *e.g.*, John, house, library, etc., while a many-worded term is composite, *e.g.*, the House of Commons, the King of England, etc.

## CHAPTER II

## INTENSION AND EXTENSION, CONNOTATION AND DENOTATION, CONNOTATIVE AND NON-CONNOTATIVE TERMS

### The Meaning of Intension and Extension

Logicians have used the terms intension, connotation, intent, comprehension, depth, implication and force, all in practically the same sense. Similarly they have regarded the words extension, denotation, extent, sphere, breadth, application and scope as synonymous. The terms that are in most general use in this connexion are intension and connotation for one thing, and extension and denotation for the other. Before Mill the words intension and extension were in vogue, but Mill preferred connotation and denotation for their affinity to the corresponding verbs to 'connote' and 'denote'.

A term denotes things and connotes attributes.

The meaning of every general concrete name has two aspects. On the one hand it indicates a number of objects, on the other it implies certain attributes that are common to those objects. These two aspects of the meaning of a name may be referred to by the employment of the words *Extension* and *Intension*, or *Denotation* and *Connotation*. So it is said that a term *connotes* attributes and *denotes* things.

Keynes distinguishes between conventional intension, which is connotation, subjective intension and objective intension or comprehension.

The use of the terms intension and connotation in the same sense has given rise to a good deal of confusion in logic. There has been, in consequence of this, a difference of opinion among logicians as to whether certain terms are connotative or not. Keynes uses the term intension in its general implicational sense, and points out that a distinction should be drawn between the various senses in which the term 'intension' has been used by logigians. A name, he points out, may have

*conventional intension,* which is synonymous with *connotation* in the sense in which the term connotation has been used by Mill. Besides conventional intension, a name may have *subjective intension* as well as *objective intension* or *comprehension.* It is necessary that terms should have fixed meaning and their meaning be fixed by convention or agreement. According to Welton, if the intension of a term is not fixed, language will lose its capacity for communicating anything like exact thought. So Bosanquet says: "Surely the question for logic is never what a name means for you or me, but always what it ought to mean."

The necessity of fixing the meaning of a term by convention.

What then do we mean by conventional intension or connotation, subjective intension and objective intension or comprehension? Those attributes which are, by agreement, taken to determine the application of a name to a large number of objects in the same sense form the *conventional intension* or *connotation* of that name. Thus animality and rationality are the conventional intension or connotation of the term 'man', since these attributes have been conventionally accepted as the attributes determining the employment of the term 'man'. *Subjective intension,* however, unlike conventional intension, is not fixed, but variable. A term may suggest a different set of ideas to different men or to the same man at different times. Thus the term 'rose' may suggest to an ordinary man a set of ideas or attributes different from those which the same term will suggest to a botanist or a lover. The subjective intension of a term is its intension as it appears to a particular subject or a group of subjects. From this point of view even a proper name has (subjective) intension, because it is associated with certain attributes in the mind of those persons who are acquainted with the object denoted by the name. Thus to a mother the name of her son suggests various attributes. We may further draw a distinction between

The distinction between conventional intension, subjective intension, and objective intension explained by illustration.

conventional intension and subjective intension on the one hand and *objective intension* or *comprehension* on the other. The objective intension of a term consists of all those attributes, known and unknown, which are possessed in common by a number of things denoted by the same name. Thus besides animality and rationality, men have many other common attributes, such as corporeality, the capacity to move, the various animal functions, the peculiar human form, etc.; all these attributes possessed in common by men make up the objective intension or comprehension of the term 'man'. We may note that from the logical point of view subjective intension is not important.

The above distinction recognised by other Logicians.

Sigwart's distinction between empirical, metaphysical and logical concepts corresponds to Keynes's distinction between subjective intension, objective intension and conventional intension. Bosanquet's distinction between the objective reference of a name and its content for the individual mind corresponds to the distinction between conventional intension and subjective intension. Logicians have made use of the term 'connotation' indifferently in the sense of subjective intension, objective intension, and conventional intension, thus giving rise to a good deal of unnecessary controversy as to the exact sense of the term regarding certain logical questions.

Distinction between extension, denotation and exemplification.

A distinction may also be drawn between extension and denotation. By the *extension* or *subjective extension,* as Keynes puts it, of a name we mean all the things, real or fictitious, to which the name may be applied. Thus the subjective extension of the term 'man' signifies all the things, real or fictitious (such as the characters of a fiction, etc.), to which the name may be applied. But the *denotation* of a term consists of all those real things to which the name may be applied. Thus the denotation of the term 'man' consists of all human beings, dead, living and to be born, to which the name 'man' is applicable.

If this distinction is borne in mind we shall understand that such terms as 'centaur', 'demi-god', 'winged horse' have no denotation, though they have subjective extension. *The exemplifications* of a term are not co-extensive with its denotation. To exemplify the term 'man' we need only adduce a few typical instances such as Ram, Shyam, Jadu, etc. Exemplification thus excludes many individuals to which the name in question may be properly applied. The denotation or objective extension of a term always has reference to a particular universe of discourse.

Keynes makes the above distinctions clear by the following apposite example. A metal is defined as an element which may replace hydrogen in an acid and form a salt. This definition of metal constitutes the connotation of the term metal. All those things, such as gold, silver, iron, copper, brass, etc., which have the above property constitute the denotation of the term metal. Thus the denotation of a term is determined by its connotation. The comprehension of the term 'metal' consists of its connotation together with those other properties that are also common to all metals, such as fusibility, metallic lustre, a high degree of opacity and the property of being good conductors of heat and electricity, etc. We may however exemplify metals by citing only a few instances, such as silver, iron and copper, which possess all the attributes common to metals. Here silver, iron and copper are exemplifications of the term metal. Similarly a triangle is defined as a plane figure bounded by three straight lines. This definition of a triangle constitutes its connotation. The denotation of the term triangle signifies all those figures to which the name triangle is applicable. Its comprehension consists of such properties as being a plane figure bounded by three straight lines, having three angles which are together equal to two right angles, and with every angle less than two right angles. We may exemplify triangle by pointing to a few triangular figures. We may note that whether we begin with the connotation or with exemplifications of a term we can arrive at its comprehension and denotation.

## Inverse Relation between Denotation and Connotation

The statement that the denotation and connotation of a term vary in an inverse ratio to each other cannot be accepted in its mathematical form.

We are now in a position to understand how far the statement that the denotation and connotation of a term vary in inverse ratio can be maintained. This doctrine means that if the connotation of a term is increased or decreased, its denotation decreases or increases in proportion, and if its denotation is increased or decreased, its connotation decreases or increases in a like manner This statement cannot be accepted in its mathematical form. We can speak of doubling or halving the denotation of a term, but it is nonsense to speak of halving or doubling its connotation. Bosanquet points out that when we add to the connotation of the term 'man' the attribute 'white', its denotation decreases much less than when we add to the connotation of the term 'man' the attribute 'red-haired', since there are many more white men than red-haired men. If this is true then with the increase of the connotation of a term its denotation does not decrease proportionately.

Within certain limits the denotation and connotation of a term vary inversely.

If we leave aside mathematical accuracy, the proposition that the denotation and connotation of a term vary inversely is found to hold good within certain limits. If we increase the connotation of the term 'triangle' by adding to it the property of having three sides equal, its denotation would diminish, as, in that case we should have to leave out those triangles, scalene and isosceles, that do not have three equal sides. Again if we increase the connotation of the term 'mountain' by adding the property 'Asiatic,' we restrict its denotation, by excluding all non-Asiatic mountains from the group. Similarly the denotation of 'white man' is less than the denotation of 'man'. Again if we increase the denotation of the term 'man' by admitting into the class all the lower animals, its connotation is naturally reduced to mere animality from the full connotation of 'man',

which is animality and rationality. What we actually have in that case is the term 'animal' instead of the term 'man'. Again if we enhance the denotation of the term 'equilateral triangle' by adding to it all other triangles which are not equilateral, its connotation diminishes, as, in that case, we have to omit the attribute of having three equal sides. Again if we enhance the denotation of the term 'Indian' by adding to it all other men who are not of Indian nationality or domicile, we diminish its connotation by omitting the attribute of having Indian nationality. We then have the term 'man' in place of the original term 'Indian'. In the same way if we decrease the connotation or denotation of a term, its denotation or connotation will respectively increase. If we take away the attribute of rationality from the connotation of the term 'man', its denotation will increase, because the denotation of 'animal' is greater than the denotation of 'man'. Similarly if we decrease the denotation of 'triangle' by excluding isosceles and scalene triangles, we shall have only equilateral triangles, the connotation of which consists of the attribute of being a plane figure bounded by three straight lines and having three equal sides. But the connotation of the term 'triangle' is merely the attribute of being a plane figure bounded by three straight lines. If we take the three terms 'animal', 'men' and 'Indian', we shall find that the first one has the widest denotation but the least connotation, and the last one, the greatest connotation but the narrowest denotation. The term 'animal' has a wider denotation but less connotation than the term 'man'. Similarly the denotation of the term 'man' is wider than that of 'Indian', but its connotation is less than that of the latter term. The expression that the denotation and the connotation of a term vary inversely thus means that, if denotation increases connotation decreases; if denotation decreases connotation increases; if connotation increases denotation decreases; if connotation decreases denotation increases.

But it is not always true that if we increase the connotation or denotation of a term, its denotation or connotation would decrease. Thus if we increase the connotation of 'triangle' which consists of the attribute of being a plane figure bounded by three straight lines, by adding the attribute of having the sum of its three angles equal to two right angles, its denotation does not diminish; for it is an attribute shared in common by all triangles. So also if we enhance the connotation of the term 'man' by the addition of the attribute 'mortality,' its denotation does not decrease, simply because all men are mortal. Again if the denotation of 'river' increases by the birth of new rivers, its connotation does not diminish, because all rivers should share in the essential riverine attributes before being regarded as rivers. The birth or death of a child does not, in a like manner, decrease or increase the connotation of the term 'man'. To limit the proposition that the denotation and connotation of a term vary inversely, Welton states, "The idea of an opposite variation of connotation and denotation is only applicable to classes which can be arranged in a series of varying generality, so that each smaller class forms a part of the next larger; such as figure, plane rectilineal figure, plane triangle, plane isosceles triangle."

The limits of the law of variation.

In the light of Keynes's distinction between comprehension and connotation and between denotation and exemplification we may understand the following propositions which he proposes in order to amend the traditional doctrine that the connotation and denotation of a term vary inversely. (1) "Let connotation be supposed arbitrarily fixed, and used to determine denotation in some assigned universe of discourse. Then it will not be true that connotation and denotation will necessarily vary inversely." (2) "If the connotation of a term is arbitrarily enlarged or restricted, the denotation in an assigned universe of discourse will

Keynes's four laws determining the relation of inverse variation between the intension and extension of a term, which amend the traditional view.

either remain unaltered or will change in the opposite direction." (3) "If the exemplification (extensive definition) of a term is arbitrarily enlarged or restricted, the comprehension in an assigned universe of discourse will either remain unaltered or will change in the opposite direction." (4) "Any arbitrary alteration in either intensive definition or extensive definition which results in an alteration of either denotation or comprehension will also result in an alteration in the opposite direction of the other." (By intensive definition Keynes means connotation and by extensive definition exemplification.)

## Connotative and Non-connotative Terms

The division of terms into *connotative* and *non-connotative* goes deepest into the nature of language, because the essence of every name is its significance. "A non-connotative term is one which signifies a subject only or an attribute only. A connotative term is one which denotes a subject and implies an attribute" (Mill). Before we consider which terms are connotative and which are non-connotative, we must remember that every concrete term has subjective intension or comprehension, though it may lack connotation, that is, fixed content or meaning. We must also bear in mind that the denotation of a term, like its connotation, is a part of its signification.

Mill's definition of connotative and non-connotative names.

Following the above we may now examine which terms are connotative and which are not. Every general name, whether concrete or abstract, is connotative. Thus the term 'tree' applies to a number of things and implies certain attributes to the possession of which the applicability of the term 'tree' is due. Similarly the term 'army', which is a general collective name, is connotative because it can be applied to a number of groups and also implies certain attributes which every army possesses. All adjectives are connotative. The term 'white' applies to a number of white things or objects such as milk, snow, etc., and

connotes the attribute of whiteness. Similarly good, bad, green, yellow, etc., are connotative terms. Though according to Jevons all abstract names are non-connotative, we may reasonably regard general abstract names as connotative. The term 'virtue' denotes a number of good qualities such as justice, veracity, benevolence, kindness, courage, temperance, etc., and it connotes the attribute of goodness. Similarly the term 'colour' denotes redness, greenness, whiteness, etc., and connotes the power of affecting the eye in a particular way. So also the term 'figure' indicates different figures such as the triangle, square, circle, etc., and connotes the attribute of possessing shape and extension. All significant singular names, such as 'the present Speaker of the House of Commons', 'the present President of the Indian National Congress', 'the Calcutta University Library', etc., are connotative. Certain proper names that have ceased to be proper names by acquiring some general sense or other are connotative: thus when we speak of a Solon, a Cæsar, a Napoleon, a Daniel, we use them in a general sense, and so these terms have both denotation and connotation. According to Mill such terms as the sun, the moon, the earth, the solar system, God, etc., are connotative.

The terms that fall mid-way between connotative and non-connotative names.

But according to Bosanquet such terms are equivalent to proper names, as their purpose is merely to indicate some object or other and not to imply any attribute. We may however say that these terms fall mid-way between connotative and non-connotative names. Such terms as 'the French revolution,' 'the falls of Niagara,' etc., may also be similarly treated.

There are certain terms which can be properly regarded as non-connotative, since they lack either denotation or connotation.

Certain terms non-connotative.

Singular abstract names such as whiteness, length, visibility, triangularity, humanity, etc., denote, according to Welton, attributes only and do not connote anything, and may properly be regarded

as non-connotative. Again such terms as centaur, winged horse, the king of France in 1930, etc., imply attributes but have no denotation as they cannot be applied to any really existing objects. They have subjective extension but no denotation.

Discussion as to why proper names should be regarded as non-connotative

Are proper names non-connotative? Logicians have differed in their opinion as to whether proper names have connotation or not. This is due to the failure to distinguish between subjective intension, connotation and comprehension Proper names certainly have subjective intension, because such a term as 'John' or 'Ram' may suggest various attributes to those who are acquainted with John or Ram. The term 'John' or 'Ram' has comprehension as well, because the object indicated by it does possesss attributes known and unknown. Yet every proper name is non-connotative because it lacks fixed content, that is, conventional intension. But Jevons argues that proper names have connotation, and that such a term as 'John Smith' implies 'Teuton' and 'male'. But Keynes points out that the term 'John Smith' may be the name of a race-horse, and it may even be the pseudonym of a woman, just as George Elliot was. The name Jacob originally meant a supplanter, but a man may be called Jacob even if he lacks that attribute. Bosanquet argues that Christian names imply sex, and surnames, descent and family relationship. But he admits here that it is self-contradictory to hold that proper names, which have individual reference, have content, which is universal. So according to him the purpose of a proper name is applicative and not implicative. It has no fixed content, he admits, and therefore we can regard it as non-connotative, though it has both subjective intension and comprehension. Even surnames are no longer connotative, because they can be changed at will.

Dr. P. K. Roy says that a proper name may originally be non-connotative, but afterwards it may become connotative. The name 'Calcutta' is non-connotative to one who does not know the

city, but it becomes connotative to him when he becomes acquainted with 'Calcutta'. This view cannot be accepted. Dr. Roy confuses subjective intension with connotation. The main point is whether the term 'Calcutta' or any other proper name has a fixed content or not. Different places may bear the name 'Calcutta' and it may be the name even of a ship.

## CHAPTER III

## THE CATEGORIES OF ARISTOTLE AND THE DOCTRINE OF PREDICABLES. VERBAL OR ANALYTIC, REAL OR SYNTHETIC, AND FORMAL JUDGMENTS

### The Doctrine of Categories

To Aristotle we owe the doctrine of *Categories* or *Predicaments* and of *Predicables.* The Aristotelian categories are ten in number and according to him they are a classification of beings. But logicians have differed as to the nature of this classification. According to Mill the classification of categories was intended by Aristotle and the Schoolmen to give us a list of nameable things. According to Bain this classification is not a classification of things but is a classification of predicates. According to Mansel it is a grammatical classification based upon parts of speech such as substantive, adjective, verb and adverb. According to Baynes it is a metaphysical classification, that is, a classification of the aspects of real beings. Welton regards this classification as one of relations conceived by the mind to interpret reality. Thus we find that there is a good deal of controversy as to why Aristotle classified categories and what is the nature of the classification. Joseph appears to be right when he says that "the categories present a logical, but they present also a real distinction, that is, a distinction in the nature of the reality about which we think as well as in our manner of thinking about it"[1].

Different views as to the nature of Aristotle's classification of categories. What really is the nature of Aristotle's classification?

Categories, therefore, are a classification of things as well as of predicates. This will become clear in the light of subsequent discussion.

Aristotle gives us a list of *ten categories,* that is, a list of things and properties, which can be predicated of an individual subject. There is a distinction recognised by Aristotle and the Schoolmen between the *essential* and *accidental* properties of a thing—to be Socrates is not necessarily to be condemned to death, but to be Socrates is necessarily to be a man. From such reflections the distinction between categories arose. We may now explain the ten categories, with illustrations which will throw light upon what has been said above. (1) The first category is *Substance*. Aristotle distinguishes between *first* substance, which is an *individual,* such as Plato, and *second* substance, which is a *class,* such as man. The second substance stands for the essential nature of a thing and is a predicate. It is what some logicians describe as a class-name. Substance therefore indicates the essential nature of a thing but attributes do not. All other categories presuppose or are incidental to substance. The proposition 'Socrates is a man' has for its predicate substance, since it gives us the essential nature of Socrates. Socrates cannot but be a man. (2) The second category is named by Aristotle *Quantity ; e.g.,* Socrates is five feet in height. Quantity is the measurable amount of a thing—five seers in weight, thirty miles away, etc., are terms which give us quantity. (3) The third category or predicament is *Quality*. We describe a thing by mentioning its qualities: *e.g.,* Socrates is ugly. Such terms as fair, good, white, beautiful, etc., belong to the category of quality, (4) *Relation* gives us the relation of one thing to another thing ; *e.g.,* Socrates is the teacher of Plato. Father, son, wife, husband, like, unlike, cause, effect, etc., are terms which belong to the category of relation. We have an example of the

Explanation of Aristotelian categories by illustrations.

category of relation when a relative term is used as a predicate. (5) The next category is *Action.* It tells us how a thing *behaves*, *e.g.*, Socrates is talking. Cutting, running, fighting, etc., illustrate the category of action. (6) The sixth category is *Passion.* This is the opposite of action and tells us how a thing is *acted upon*, *e.g.*, Socrates is condemned to death. To be hated, to be killed, to be thrown, etc., give us the category of passion. (7) *Place* is another category. It tells us *where* a thing is; *e.g.*, Socrates is in his room. Here, there, etc., are included in the category of place. (8) *Time* is the next category. It tells us *when* a thing is; *e.g.*, Socrates is talking to young men at midday. Now, then, at dawn, in the evening, etc., fall under the category of time. (9) We have as our next category *Situation.* It tells us *how* a thing is *placed* relatively to another; *e.g.*, Socrates is lying on his bed or sitting in his chair. Such terms as upside down, horizontal, etc., also illustrate the category of situation. (10) The last category is *State*. 'Socrates is dressed' gives us an example of this category. It tells us *in what condition* a thing is. To be armed, to be shod, etc., are examples of this category.

We need not critically examine Aristotle's doctrine of categories, nor need we consider other lists of categories provided by subsequent logicians, such as Mill, Kant, and others. We may therefore close the discussion of categories here.

## The Doctrine of Predicables

Categories and predicables.

"Aristotle's list of categories was concerned with a classification of things independent of any relation in which they may stand to other things, while his list of *predicables* was concerned with a classification of attributes as related to a subject." (Welton and Monahan). If we understand the meaning of a

term we can at once say to which category it should be referred. We know that the term 'beautiful' belongs to the category of quality whether it is related to any definite subject or not. But we cannot determine to what predicable a term belongs without knowing its relation to some given subject. Thus predicables are a classification of terms as determined by their relation to some given subject.

The list of predicables given by Aristotle and Porphyry.

Aristotle provides a list of five predicables, *viz.*, Genus, Definition, Differentia, Proprium and Accidens. The traditional list of predicables is due to Porphyry, whose classification differs somewhat from that of Aristotle. According to him the predicables are genus, species, differentia, proprium and accidens (accidens may be either separable or inseparable). We need not consider Aristotle's list of predicables in detail as our purpose is mainly to explain Porphyry's list of predicables. We may however explain what Aristotle means by 'definition' which is not included in Porphyry's list. When the predicate of a proposition states the connotation of the subject, we have a predicate which is a definition in relation to the subject. In the proposition "man is a rational animal," the predicate "a rational animal" is a definition in relation to the subject 'man', because the connotation of 'man' consists of animality and rationality. A *genus* is a class which has under it sub-classes which are its *species.* Thus under the genus 'animal' there are sub-classes or species such as man, dog, horse, etc. The denotation of a genus is wider than that of its species; but the connotation of a species is greater than that of its genus. So it is said that a genus includes the species in denotation, while a species includes the genus in connotation. A term which is a genus in relation to one term may be a species in relation to another term. Thus 'triangle' is a genus in relation to the terms denoting equilateral, isosceles

and scalene triangles, but it is a species in relation to the term 'rectilineal figure'. Again 'animal' is a species in relation to 'living objects,' but it is a genus in relation to men, monkeys, elephants, etc. Thus the term genus and species are correlative terms since the meaning of one can be understood only by understanding the meaning of the other. Species which are co-ordinate and belong to the same genus are called *cognate species.* Thus in one of the above examples men, monkeys, elephants and in the other equilateral, isosceles and scalene triangles are cognate or co-ordinate species. Every genus which is above a given species is a cognate genus. Thus each of the terms 'triangle' and 'rectilineal figure', is a cognate genus of each of the terms 'equilateral triangle,' 'isosceles triangle and 'scalene triangle'. The next higher class of a species is called its *proximum* or *proximate* genus. Thus 'triangle' is the proximum genus of 'equilateral triangle' and 'animal' of 'man'. Every genus or species which can function both as a genus and as a species is called a *subaltern genus or species.* Thus the term 'animal' which is a genus in relation to 'man' and a species in relation to 'living things,' may be called either a subaltern genus or a subaltern species. A *summum genus* is a class which cannot be subordinated to any other higher class, that is, a summum genus can never be a species. An *infima species* is a class which cannot be sub-divided into subordinate classes, but can only be divided into individuals, that is, an infima species can never be a genus. Man, though it may be subdivided into mathematicians, poets, politicians, etc., is an infima species, because these sub-divisions are not natural classes and therefore are not species. The division of man even into male and female is not regarded by traditional thinkers as a division into natural sub-classes. Man can be divided naturally only into individuals such as Plato, Socrates, Aristotle, and others. To state the character of a natural class or a real kind, that is, of a genus or species, Mill says, "every class which is a real kind, that is, which is distin-

guished from all other classes by an indeterminate multitude of properties not derivable from one another, is either a genus or a species". Aristotle recognised ten summa genera, which are his categories. Modern logicians recognise but one summum genus, viz., substance. But for practical purposes we may recognise several summa genera, such as 'man' for *Sociology*, 'material substance' for *Chemistry* and 'life' for *Biology*, and so on.

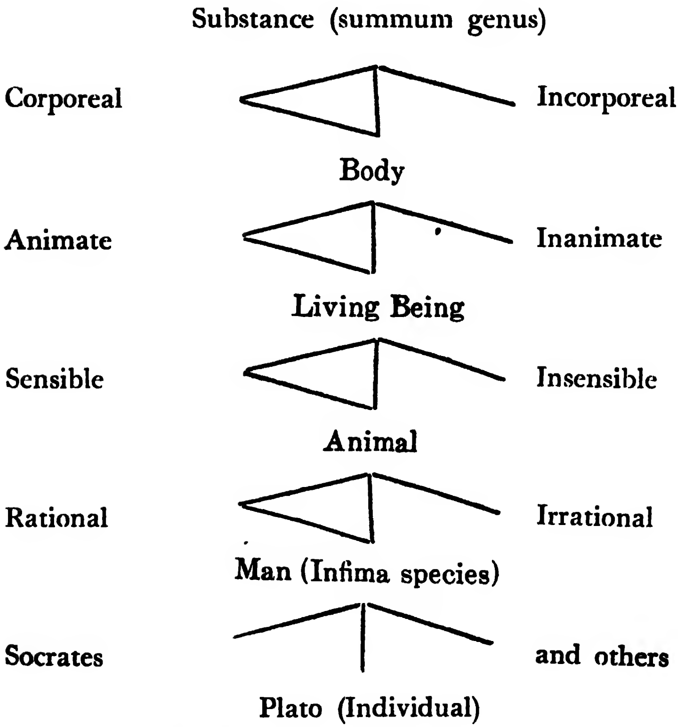
*Differentia* is the peculiar attribute of a species which differentiates it from other things or cognate species. It is the excess of connotation of a species over its proximate genus. Thus nationality is the differentia of 'man' by which human beings are distinguished from other co-ordinate species such as monkeys, elephants, sheep, etc., and this rationality is the excess of connotation of 'man' over its proximate genus 'animal'. Similarly 'having three equal sides' is the differentia of the equilateral triangle.

A *property* (proprium) follows from the definition of the subject, that is, from the connotation of a term, just as an effect follows from its cause or a conclusion follows from its premises. The connotation of man gives us two attributes, *viz.*, animality and rationality: the former speaks of the general nature of man and the latter of his specific nature. From animality follow the properties mortality, corporeity, etc. Similarly from rationality follow such properties as the power of foresight, the power of calculation, the ability to use tools, etc. While a property follows from the essential nature of a thing and is necessary to it, *accidens* (accidentia) are not indispensable to a thing and do not follow from its essential nature. Thus a fair complexion, honesty, beauty, etc., are accidental qualities of man. A distinction may be made between separable and inseparable accidens. That Socrates is a Greek, is an inseparable accidens of Socrates; but that he is a teacher is a separable accidens of Socrates.

When a general term is predicated of another general term, the predicate is a genus. Thus in the proposition, 'Man is an animal' the general term 'animal' is predicated of the general term 'man'; hence the predicate 'animal' is the genus here. So also in the proposition 'A triangle is a rectilinear figure', the predicate 'rectilinear figure' is the genus. When a general term is predicated of an individual term we have a species as the predicate, as in the proposition, 'Socrates is a man,' the predicate 'man' is the species to which the individual named Socrates belongs. "The excess of the connotation of a species over that of its proximate genus is called the differentia or difference of that species" (Welton). Thus in the judgment 'man is rational' we have as predicate the differentia of 'man', that is, the attribute which distinguishes man from other cognate species. Again in the proposition 'isosceles triangles have two equal sides,' we have a differentia as the predicate. The distinction between differentia and property (proprium) is founded upon conventions of language rather than on the nature of things. We have already stated that a property follows from the essential nature of a thing. Thus in the propositions 'man is mortal', 'man is a cooking animal', the predicates are differentias in relation to the subject. In this regard Porphyry is at one with Aristotle. An accident can be removed from a class or an individual without affecting it fundamentally. Here also Porphyry's account of accidents is not different from that of Aristotle. But the former makes a distinction between separable and inseparable accidents. In the propositions 'All European ruminants are cloven-footed', 'Gandhi is an Indian,' 'Socrates had a snub-nose', 'Alexander is the son of Philip', the predicates are inseparable accidents. In the propositions 'some men are red-haired', 'Pandit Nehru is the Prime Minister of India', we have separable accidents as predicates. We may mention that in the proposition 'some animals are men' the predicate in relation to the subject is a species.

Explanation of Prophyry's list of predicables by examples.

We may illustrate the relation between genus and species by the *Tree of Porphyry* or Ramean Tree (so named after Ramus who gave prominence to it):—




## Verbal, Real and Formal Judgments

**The nature of verbal, analytic or essential propositions.**

It will be convenient at this stage to treat of the distinction between verbal and real propositions or judgments, and also to explain what a formal proposition is. The terms *verbal, analytic*, and *essential* are often used in the same sense, while the terms *real, synthetic, accidental* and *ampliative* are synonymously used when they are employed to qualify proposi-

tions. The distinction between the analytic and synthetic or the verbal and real applies only to affirmative propositions. A purely verbal proposition is a synonymous one, and cannot be regarded as the same as an analytic proposition, in which the predicate asserts some essential attribute or attributes of the subject. Such propositions as 'Cicero is Tully' and 'An ass is a donkey', 'Wealth is riches', etc., are purely verbal propositions. But the term verbal proposition has been used by logicians to denote essential or analytic proposition as well. We have already noted the distinction between the essential attributes of a thing and its accidental attributes. An essential, analytic or verbal proposition is one in which the predicate asserts some essential attribute or connotation of the subject. Thus the propositions 'man is rational', 'man is a rational animal', 'man is an animal' are essential propositions because in each case the predicate asserts some essential attribute or attributes of the thing denoted by the subject term. Similarly the proposition that 'a square is a rectilinear figure bounded by four equal sides' is a verbal proposition. In other words, an essential analytic, or verbal proposition asserts some attribute, or all the attributes, connoted by the subject term. According to Mill such a proposition is useless because it does not impart any new information about the subject and is not capable of proof. As soon as we know the subject term we know its essence or connotation and we require no proposition to state it. It has therefore been stated that the predicate of an analytic or essential proposition must either be genus or definition or differentia. But we may here remark that because proper names are non-connotative such a proposition as 'Socrates is a rational animal' should not be regarded as an analytic proposition. The Aristotelians regard it in this way, because Socrates has no connotation and therefore cannot be defined. Such a proposition should properly be regarded as an accidental or real proposition since the predicate in this case states something which is not implied by the subject term; but when the subject is a significant singular term we may have an analytic proposition, as in the case 'this great philosopher is a

rational animal', because to be a philosopher is to be a rational animal, that is, the subject implies the predicate. Thus "a verbal proposition is one which gives information only in regard to the meaning or application of the term which constitutes its subject," as Keynes puts it.

The character of a real, accidental or synthetic proposition.

While an analytic proposition affirms some part or the whole of the connotation of the subject, a real, synthetic or accidental proposition affirms attributes accidental to the subject, and not essential to it. Thus such propositions as 'men are cooking animals', 'some men are red-haired', 'some Indians are industrious', 'all men are mortal'. etc., are accidental or synthetic propositions. Such propositions were called accidental by the Schoolmen because they assert some attribute not essential to the subject, and hence, accidental to it. So in a real or synthetic proposition the predicate in relation to the subject is either a property (proprium) or an accident.

The nature of a formal proposition.

The formal judgment or proposition is one the truth of which can be estimated by considering the form of it. Such propositions as A is A, No X is not-X, Any X is either P or Q, If all A is B then no not-B is A, A man is a man, etc., are formal judgments or propositions. A formal judgment should be distinguished from a verbal judgment because the validity of a formal judgment depends upon its formality and not upon its material content. The validity of a real proposition depends upon questions of fact, that of a verbal proposition upon mere meaning, while the truth of a formal proposition depends upon its bare form. From the logical point of view formal propositions are very important.

## CHAPTER IV

# THE PROBLEM OF DEFINITION

General nature of Definition.

*Definition* consists in unfolding the connotation of a term. We define a term so as to make its meaning clear and the progress of thought possible. In every scientific discussion it is necessary to define all the important terms that are used, in a clear and distinct way for the proper understanding of the subject. Thus geometry at the very outset defines such terms as point, line, triangle, circle, etc. Similarly the physical sciences define such terms as element, atom, energy, motion, acid, salt, and so on.

It has been a subject of controversy whether definition is of names or of things. To us this controversy is meaningless. Names are very often names of real things, and in defining such names we cannot but take cognisance of the fundamental attributes which are connoted by the name. Thus when we define 'man' as 'a rational animal' we not only define the name or term 'man' but also define the things denoted by the term. But in some cases the object of definition is merely to make the meaning of a name clear without any reference to real things; such definitions are definitions of names only. Definitions of such terms as 'fairy', 'nymph', 'centaur', 'ghost', etc., are nominal definitions because they are not known to denote real things. Mill argues that geometrical definitions are ideal and not real. In the real world there cannot be any geometrical point that has only position but no magnitude. Similarly he argues that we may rightly define a dragon as a serpent breathing flame, though no such being

Nominal and real definitions.

exists in the world. We agree with Mill that there are nominal definitions, that is, definitions of names only. But there are other definitions which are as much definitions of things as of names. In order to define names which have their objective counterpart in the world of reality we have to have recourse to observation, analysis, comparison, abstraction, generalisation, and we must also be able to distinguish between primary or fundamental qualities and secondary or derivative qualities of things, as without bearing in mind these distinctions such names cannot be well defined. So our conclusion is that some definitions are *real*, and must be based upon the nature of things; while others are *nominal*, in which case we have to depend upon usage. Mill also recognises this when he says that, though definition is of names, it enters deep into the nature of reality, as in defining justice, virtue, etc. Remembering that names are often indissolubly related to things, we may say with Mill that, "the simplest and most correct notion of a definition is, a proposition declaratory of the meaning of a word, namely, either the meaning which it bears in common acceptation, or that which the speaker or writer, for the particular purposes of his discourse, intends to annex to it." We may here observe that Ueberweg's view that all definitions are of notions is also one-sided. We define terms, and since terms stand for notions and have objective counterparts, definitions, though of terms, are as much definitions of notions as of objects. According to Welton and Monahan, real definition may be either analytic or descriptive, while a nominal definition may be either ostensive or biverbal. We shall explain hereafter the meaning of these expressions. But at this stage we may distinguish between *substantial* definition and *genetic* definition.

Substantial and genetic definition.

A term is defined *substantially* when the essential attributes of the things denoted by the term are stated; a term is defined *genetically* or *constructively* when we state how the distinction of the class denoted by it is effected. Thus in defining the term 'tree' substantially, we state

the essential attributes which are common to all trees, but we define the term genetically by describing the considerations as a result of which certain things come to be called by the name 'tree'. But genetic definition is useless for logical purposes.

Implication of the statement that definition should be per genus et differentiam.

We are now in a position to understand what is meant by the Scholastic view that definition should be *per genus et differentiam*. This means that in defining a species we must state its generic property and also its differentia. Thus we should define man as a rational animal, and isosceles triangle as a three-sided rectilinear figure having two equal sides. But Mill holds that in defining a class-name we must state not only its generic property and differentia, but also those other attributes which are common to all the members of the class denoted by the name. Thus according to him properties and inseparable accidents should also be included in definition. Thus he says that man should also be defined as a rational, organised, corporeal and living animal. But such a definition does not observe the law of p' simony. Since life, corporeality, organisation, etc., follow from animal nature, we can rightly define man as a rational animal. So in defining a class we should not mention any property (proprium) or accident, but rest content with the statement of its essential attributes, that is, those attributes upon which others are grounded. Again, in defining a species we should state its differentia and the nature of its proximate genus and not of some remote genus. Thus to define 'man' as a rational being is untenable, since it does not state the animal nature of man. If by differentia we mean one attribute or a collection of attributes, then it is sufficient to say that definition should be per genus et differentiam, instead of saying with Mill that it should be per genus et differentias. Every definition of a general name must be a universal proposition, that is, it must be expressed in the form 'Every S is P'.

A scientific definition should be *analytic,* that is, it should state the connotation of the term defined. Thus the scientific definition of a triangle is that it is a rectilinear figure bounded by three straight lines. But various kinds of unscientific or popular definition are recognised. (a) A name whose meaning is unknown is *ostensively* defined by pointing out the thing which it indicates. Thus, pointing to an orange we may say, 'This is an orange'. Recourse is often had to such ostensive definition in the education of children. But this is not definition proper. (b) When the meaning of a name is unknown we may define it by a synonym of known meaning. Commiseration may be defined as pity, or Swaraj as self-government. In translating from a foreign tongue we may be said thus to define *biverbally.* (c) An *extensive* definition consists in defining a term by pointing out certain things denoted by it. Thus to define virtue we may point out that pity and veracity are virtues, or to define mountain we may say that the Himalayas, the Vindhyas, the Alps are mountains. (d) *Descriptive* definition consists in describing a thing by means of some of its unimportant attributes. Thus we describe a landscape by naming some of its features, or a horse by saying that it is an animal beautiful to look at, has four legs and is used in time of war or for drawing carriages. Of the above kinds of definition logic is concerned with analytic definition only, since it greatly contributes to the advancement of knowledge and is based upon the knowledge of the nature of things. Analytic definition is definition per genus et differentiam. Though analytic definition alone is scientific, yet for the progress of knowledge we often have recourse to description, especially when the connotation of a term is unknown. Description gives us information about the minor attributes of things.

**Different kinds of definition, of which the analytical definition alone is scientific.**

The Scholastic view of definition that it should be per genus et differentiam, is no doubt important. But what we regard as

Some remarks on the Scholastic view of definition that it should be per genus et differentiam.

the differentia of a species may, in course of time, come to be regarded as a mere property of it. Thus properties may become differentiae and differentiae properties. Thus with the growth of knowledge, definitions undergo modifications. Discovery and definition should therefore go hand in hand. So no definition of class-names should be regarded as unalterably fixed. Though the object of definition is to clarify the meaning of a term, it should not bar the progress of thought and science. Thus with the progress of knowledge, the denotation of a term may increase and its connotation consequently may become vague. But even in such a case an attempt should be made to define properly after due observation. There may be marginal instances between classes, but the attempt should be made to bring them under one class or another. Thus the sponge, which was supposed to be a marginal instance between animals and plants, has been found to be akin to animals and has been included under the class 'animal'. Furthermore, marginal instances should not prevent us from defining a class if such definition serves our purpose.

Limits of definition.

We may now point out the limits of definition. Not all names can be defined: only significant names are capable of definition. Proper names, therefore, which have no connotation, cannot be defined. Similarly, elementary sensations which cannot be analysed can have no definition. Thus feelings of pleasure, pain and anger cannot be defined. We cannot define the sensation of white or red, nor can we define the sensation of sweetness. Singular abstract names, such as triangularity, whiteness, etc., cannot be defined because such terms are not significant. We can however describe how a particular sensation originates by the stimulation of a sense organ. We can also point out what are the constituent elementary feelings in a complex feeling. But such processes cannot be regarded as definition

proper. But a singular significant name can be defined by marking out its fundamental character and omitting inessential attributes. Thus 'the present President of the Indian National Congress' may be defined as the person who is at present the executive head of the Congress. We need not name the other minor attributes which this person possesses. According to the Schoolmen, summa genera cannot properly be defined, since they cannot be brought under higher genera. Thus according to them, the Aristotelian categories cannot be defined. Again according to them, sub-divisions of an infima species cannot be defined. Thus the term 'negro' cannot be defined. Modern logicians however hold that the term 'negro' can properly be defined as a black man. Similarly such terms as Europeans or Indians may be defined. General abstract terms may also be defined. Thus 'fault' may be defined, according to Mill, as a quality productive of evil or inconvenience, and 'eloquence' as the power of influencing the feelings by speech or writing. Definition is the statement of the connotation of a term, and as such every term that has connotation can be defined and no term that is devoid of connotation can be defined.

### The Rules of Scientific Definition

A number of rules are given by logicians to guide us in the act of properly defining a term. They are the following:—

The violation of the first rule leads to either redundant or too narrow or too wide definition.

(1) Definition must give only the essence or connotation of the term defined. "The essence of anything is that in virtue of which it is such a thing. It is in virtue of being a three-sided rectilinear figure that anything is a rectilinear triangle: in virtue of being an institution for the education of the young, that anything is a school: in virtue of having value in exchange, that anything is wealth" (Joseph). If the definition states more than the connotation of the term to be defined, the definition becomes *redundant* and may be *too narrow*. A definition is too narrow

when the denotation of the definition is less than the denotation of the term to be defined. Thus if a triangle is defined as a plane rectilinear figure bounded by three equal straight lines, the definition becomes too narrow, because it excludes isosceles and scalene triangles. Again to define man as a rational animal with a white complexion is to leave out the so-called 'coloured peoples,' and the definition thus becomes too narrow. But if properties or inseparable accidents are included in the definition, the denotation of the definition remains the same as that of the term defined. Thus if we define a triangle as a three-sided rectilinear figure having three angles, we add to the connotation of the term the property of having three angles, a property which all triangles possess. In this case the denotation of the term defined is the same as that of the definition. Yet such a definition is faulty: it is redundant or superfluous, because it does not observe the principle of parsimony, and also because it seems wrongly to suggest that the property in question is not really a property but a part of the full connotation of the term—it is either a generic attribute or a specific attribute of the thing defined. If we omit some part of the connotation of a term in the definition, the definition is *incomplete* and becomes *too wide*, for then the denotation of the definition becomes wider than the denotation of the term defined. Thus if we define a triangle merely as a rectilinear figure, we mention only a part of the connotation of the term triangle, and thus the denotation of the definition, since it includes figures other than triangles, becomes wider than the denotation of the term defined. Similarly the definition of man as an animal is too wide. This rule follows from the nature of definition, which should be per genus et differentiam. From the above it follows that the denotation of a definition should be equal to the denotation of the term defined. (2) Definition should be clear and should not be expressed in unfamiliar, figurative or ambiguous language. The non-observance of this rule virtually leads to an explanation

**The violation of the second rule leads to obscure definition.**

of the unknown by an equally or more unknown epithet. The fallacy arising out of the breach of this rule is technically known as "*ignotum per ignotius*' or '*per aeque ignotum.*' Memory should not be defined as 'the tablet of the mind.' Again, such definitions as 'necessity is the mother of invention', 'man is the crown of creation', etc., are figurative. Dr. Johnson's definition of a net as "a reticulated fabric, decussated at regular intervals" is obscure. 'Eccentricity is peculiar idiosyncrasy,' 'fluency is an exuberance of verbosity,' are also examples of obscure definition.

The violation of the third rule leads to tautologous or circular definition.

(3) The definition of a term should not be by some synonym, that is, a term should not be defined by itself. The violation of this rule leads to *tautologous definition,* which is technically known as *circulus in definiendo.* Thus 'wealth is riches,' 'truth is veracity' are examples of tautologous definition. Again, a cause should not be defined as that which produces an effect, nor an effect as that which is produced by a cause. Correlative terms can be defined only by defining the relation between them. Again to say that 'pleasure is desired, that which is desired is good, what is good is pleasant, therefore pleasure is good' is an example of circular definition. But synonymous definition is often useful and adds to our knowledge when an obscure term is defined by a synonym which is simple and clear, *e.g.,* 'an entrepreneur is the organiser of an industry.

The violation of the fourth rule involves the fallacy of negative definition.

(4) A definition should not be negative when it might be affirmative. 'Virtue is the opposite of vice', 'solid is that which is neither liquid nor gaseous,' are examples of negative definitions, and are useless because they do not unfold the meaning of the term defined. But terms which have a negative sense can rightly be defined negatively. Thus we may define a bachelor as an unmarried man, an alien as a person who is not a citizen, or an outlaw as a person who does not receive the protection of law.

## Some concluding Remarks

In conclusion we may remark that it is often very difficult to define terms *per genus et differentiam* because we do not know the connotation of every term. Thus it is not easy to define the dog, monkey, elephant, etc. So also social sciences such as economics, politics, sociology, etc., are not easy to define. We find difficulty also in defining such terms as community, right, duty, etc. Our knowledge of things not being perfect and being progressive, we have sometimes to have recourse to descriptive definitions, which often prove useful. In such a definition we mention attributes which are not essential to things. Thus economics is defined as the science that deals with consumption, production and distribution; a nation is defined as an organised people having a common nationality and occupying definite territory, and so on. Again man is defined as a cooking animal or a featherless biped. Such definitions are descriptive. Some descriptive definitions may be very useful, others not so useful. We have also seen that definition should be as progressive as our knowledge and should not be regarded as stereotyped. Atoms were once defined as hard, .ltimate, indivisible particles of matter, but modern physical research has established that the atom is not simple but composite, being made up of protons and electrons.

# CHAPTER V

## THE DOCTRINE OF DIVISION

### Division

The general nature of division.

Keynes says, "The term Division may be defined as the setting forth of the smaller groups which are contained under the extension of a given term. It is also defined as the separation of a genus into its constituent species." Though division involves the analysis of the denotation of a term. it does not consist in enumerating the individuals belonging to a class. In logical *division* we divide a *higher class* or a *genus* into *sub-classes* or *species,* which may again be sub-divided into sub-species till the infima species are reached. The genus which is to be divided is called the *totum divisum* (divided whole) or *dividend.* The species into which it is analysed are styled the *membra dividentia* (dividing members). Thus we divide animals into men, elephants, horses, monkeys, etc., and triangles into equilateral, isosceles and scalene. In dividing a genus we think of an attribute which is possessed by some of its members but not by others, and this suggests the *fundamentum divisionis* or basis of division. In the above example of the division of triangles, the principle of division is the relation of the sides. Similarly when we divide triangles into obtuse-angled, right-angled and acute-angled, the division is according to the size of the largest angle In dividing men into Europeans, Asiatics, Africans, etc., the principle of division is the continent in which they live. The above examples show that the same genus may be divided according to different principles, as in the case of dividing triangles according to the relation of their sides and according to the largest angle. If the same genus is divided according to different principles, we have an example

of *co-division*, and the classes obtained by co-division overlap each other. Thus a right-angled triangle may be either isosceles or scalene.

Progressive division must be gradual and should not make a leap.

In *progressive division* we must proceed gradually from the higher class to the lower, and must not jump from one class to another leaving an intermediate class behind. Division must be gradual and step by step. This is expressed by saying '*Divisio non faciat saltum*', that is, division must not make a leap. Thus in dividing men we may first divide them into Asiatic, Europeans, Americans, etc., then we may sub-divide Asiatics into Indians, Chinese, Persians, Japanese, etc., next we may sub-divide Indians into Bengalees, Assamese, Oriyas, Madrasees, etc. Such a division is progressive and gradual.

The relation between division and definition. Division presupposes definition.

Division presupposes definition. If we are not aware of the definition of a name, we cannot find out the principle of division. Definition gives us the connotation of a term and it is connotation that determines denotation. Thus the definition of the triangle gives us the connotation of triangle, and in dividing we group triangles into sub-classes according to some principle, but if we are not aware of the connotation of the class 'triangle,' we cannot divide it. We may make the relation between definition and division clear by another example. If we want to divide wealth into its species, we must know the definition of wealth, namely, that it is that which has value in exchange. There are things which have value in use and not in exchange, and such things should be excluded from the category of wealth. When we have thus obtained the definition of wealth, we can divide it into its species. We therefore find that definition is fundamental, while division is derivative.

Logical division should be distinguished from physical, metaphysical and verbal division. In *physical* division we divide

Logical division should be distinguished from physical, metaphysical and verbal division.

an individual thing into its constituent parts, as in dividing a ship into mast, hull, sails, etc., or in dividing a watch into case, hands, face, etc. Chemical division, *e.g.*, the division of water into hydrogen and oxygen, is nothing but physical partition. In the case of logical division the genus divided is predicable of each of its sub-classes. Thus X is divided into Xa, Xb, Xc, etc. We can say man is an animal, the monkey is an animal, the dog is an animal, and so on. But we cannot say that the mast is a ship or the hull is a ship. Metaphysical *division* is the mental division of a thing into its attributes, as when we divide gold into yellowness, hardness, malleability, etc. Here also the thing divided is not predicable of each of its qualities. We cannot say that yellowness is gold. Again *verbal* division, as in the case of dividing an ambiguous word into its different significations, is not logical division. Thus we distinguish the meanings of 'vice' as either some moral fault or a mechanical instrument.

Division is not merely a formal process but it requires a knowledge of facts.

Some logicians wrongly regard division as merely formal. A correct division cannot ignore consideration of facts. Our knowledge of the general attribute does not give us the differentiae of the species, without the knowledge of which a higher class cannot be divided into sub-classes. This knowledge is gained by observation of facts. The definition of 'triangle' does not give us the relation of its sides according to which we divide it into equilateral, isosceles and scalene. Again the knowledge of the connotation of 'animal' does not give us the differentiae of its sub-classes, which can only be learned by a study of facts. Thus it is not correct to say that classification is material while division is purely formal. Moreover, division involves classification. If we want to divide novels, we have to enumerate mentally the novels which we have perused.

**The Fundamental Rules of Division**

Our previous study enables us now to examine the fundamental *rules of division.* They are:

I. "The members of the division shall be mutually exclusive."

II. "Collectively they shall be exactly co-extensive with the class that is divided." This means that the denotation of the sub-classes collectively should be equal to the denotation of the class divided.

III. "Each distinct act of division should proceed throughout upon one and the same basis or principle."

IV. "If the division involves more than one step, it should proceed gradually and continuously from the highest genus to the lowest species, that is to say, it should not pass suddenly from a high genus to a low species." (Keynes).

Though not strictly necessary, another rule may be added, *viz.,*—

V. No individual sub division should be equal in extent to the class divided.

Logical division is impossible without observing the first and second of these rules, which are fundamental. Rule III does not exclude the possibility of dividing a genus according to different principles. Rule IV requires that division should give us a hierarchy of classes or a graduated series. Non-observation of the first and the third rules may lead to cross or overlapping division. Thus division of animals into invertebrates, fishes, amphibians, reptiles and birds involves *cross division.* Again if we divide men into white men, negroes, yellow men, Hindus, Mohammedans, Christians, Europeans, Americans, Asiatics, Africans, etc., there is cross or *overlapping division,* because white men may be

The non-observation of the first and the third rule may lead to the fallacy of cross or overlapping division.

Europeans or Americans and Christians or non-Christians. But in some cases the non-observation of the third rule may not lead to cross division, as in dividing triangles into isosceles, scalene and equiangular, because equiangular triangles are equilateral at the same time. Again the observation of this rule may not prevent overlapping division, as when we divide triangles into equilateral, isosceles and scalene, since equilateral triangles are at the same time isosceles. To get rid of this difficulty we should define an isosceles triangle as a triangle which has only two of its sides equal.

The cases in which the division involves the fallacy of either too wide or too narrow division.

The breach of rule II leads to either *too wide or too narrow division*. If in the division of a class we omit some of its sub-classes, then the collective extent of the sub-classes becomes less than the extent of the class divided, and in such a case there is too narrow division. Thus if we divide rectilinear figures into triangles and quadrilaterals only, or animals into men, monkeys and horses only, there is too narrow division. Again if in dividing a class we mention all the sub-classes and more classes besides, then the collective extent of the sub-divisions becomes greater than the extent of the class divided, and in such a case there is too wide division. Thus if we divide men into Europeans, non-Europeans, and anthropoid apes, or coins into gold coins, silver coins, nickel coins, copper coins, and promissory notes, we have too wide division. Again if we divide teachers into school-teachers, teachers other than school-teachers, lawyers, scientists, poets, philosophers, etc., we have an example of too wide division. Even if the third rule is observed, that is, if we adhere to one principle in dividing, there may be either too narrow or too wide division, for we may omit some of the sub-classes or include some species within such sub-classes as are outside the genus divided. The non-observation of the fourth rule may also lead to too narrow division. If we leave out intermediate steps in progressive division, the collective extent of the

sub-divisions may be less than that of the class divided. Progressive division is exemplified when a botanist starts with the summa genera of plants, *viz.*, oxogens, endogens and acrogens, and sub-divides them into varying orders, which again he subdivides into varying genera and these again into varying species till the infimae species are reached. Such procedure is scientific and helps in the development of knowledge.

**Division by Dichotomy, or Bifid Division**

Division by Dichotomy is formal in character. Some observations against such divisions.

Dichotomous division is based upon the principles of contradiction and excluded middle. Division by *Dichotomy*, or dichotomy by contradiction, is the division of a class simply with reference to the presence or absence of a given attribute or set of attributes. Thus we may divide colour into white colour and not-white colour, or X into XA and XA′, A′ meaning not-A. Such a division, though formally valid, does not represent our actual procedure of division. Besides, in dividing X into XA and XA′ we require the knowledge of one of the terms in the sub-divisions, *viz.*, that XA is included in the class X. Dichotomous division is exemplified by the Tree of Porphyry or Ramean Tree. Many logicians eloquently praise division by dichotomy as perfect. Thus Bentham and Jevons are enchanted by the matchless beauty of the Ramean Tree. We however cannot be satisfied with merely formal division, in which the negative term is always indefinite. The following objections can be made against division by dichotomy. The sub-class indicated by the negative term is always indefinite in extent, though we may go on sub-dividing it. XA′ does not tell us what its denotation is. Such a division does not take into consideration actual facts and is thus cumbrous. When a class can be divided into natural sub-classes, it is meaningless to divide it by two contradictory terms. Further such a division is hypothetical in character, since we do not know whether

the class indicated by the negative term is existent or not. Mill regards such a division as too formal, while Mansel speaks of it as not sufficiently formal. From the practical point of view it is useless. No division is possible without material knowledge. It does not provide any means by which correct divisions may be effected. But it may be said that except in the case of division by dichotomy, we cannot formally find out whether a particular division conforms to the rules of division or not. For material division, it is necessary to find out whether the sub-divisions have reference to the universe of discourse in question. But in formal division such reference is not necessary, since dichotomous division is hypothetical in character, the negative term being indefinite. Welton remarks that such a division, if not purely formal, must also be based on fact, as in the case of the division of men into Europeans and non-Europeans if any, Europeans into Englishmen and non-Englishmen if any, and so on.

Purely formal division is useless.

Venn gives us various examples of formal division which for all practical purposes are useless. We may give an example. If we are concerned with three terms S, M. P, we may have the following formal divisions, *viz.*, SMP, SMP′, SM′P, SM′P′, S′MP, S′MP′, S′M′P and S′M′P′. Here S′, M′, P′ stand for not-S, not-M, not-P. For other examples of formal division students may consult Venn's *Symbolic Logic.*

Division is very useful from the scientific point of view.

Division is very useful from the scientific point of view. It clarifies our thinking and makes progress in knowledge possible. Every science has to have recourse to division and classification, without which it cannot expect to attain its object.

## CHAPTER VI

## THE DEFINITION AND NATURE OF PROPOSITION

Definition of Proposition

The problem of *proposition* in logic is so central and important that it is essential to treat it with some fulness. Logic is the science of thought; we think only when we judge; and there is a close connection, if not identity, between judgment and proposition. The judgment is the unit of thought in the same way as the sentence is the unit of language. Ignoring the distinction between judgment and proposition for the present, and regarding them as identical, we may state with Johnson that, "a systematic treatment of logic must begin by regarding the proposition as the unit from which the whole body of logical principles may be developed." According to him, "A proposition is that of which truth and falsity can be significantly predicated." Russell also regards proposition as anything which is either true or false.

The relation between judgment and proposition.

Only assertions, that is, affirmations or denials, can claim to be true, and we cannot assert without judging. So Judgment and proposition are closely related. What then is the relation between them? According to Bosanquet, "Judgment claims to be true, that is, presupposes the distinction between truth and falsity." Thus he defines judgment in the same terms in which Johnson and Russell define proposition. Should judgment and proposition then be regarded as identical? Johnson argues that this is not possible, because judgment is concerned with the mental attitude of the person judging and is subjective, or as he says, *epistemic*. He does not agree with Bosanquet and others, who hold that a proposition is the expression of a judgment in words or language, because a mere

verbal expression cannot lay claim to truth; the expression of a judgment in language is a sentence, not a proposition. So according to him judgment is wider than proposition. Every proposition, however, involves judgment, since there cannot be any assertion without judgment. Therefore he says that we pass judgment upon propositions, which stand for some fact, that is, are objective or *constitutive* in nature. Anything that is true or false must have objective reference and must also imply an assertion. But, we may ask, if proposition is regarded merely as objective, as it is by Johnson, how can it claim to be true? If however it does claim to be true, it must combine both the subjective and the objective aspects of knowledge, and therefore judgment and proposition must be inseparable. This means that a proposition is at the same time a judgment. Without fulfilling this condition, a proposition cannot claim to be true, and cannot form the central problem of logic, from which all other logical principles can be derived.

Identity between judgment and proposition.

Logicians have generally used the terms judgment and proposition in the same sense, and we can avoid confusion if we can find some clue to identify them, or at least to find a very close relation between them. One school of logicians has treated of judgments exclusively without any reference to propositions, another has treated of propositions alone without any reference to judgments. But this has led to difficulties. The former school ignores the close connection between thought and language, while the latter concerns itself mainly with the grammatical structure of proposition and its different forms. It appears to us that a compromise can be effected between Bosanquet and Johnson, since the former's definition of judgment is the same as the latter's definition of proposition. If we do not take judgment in its wide and psychological sense, but narrow its meaning for logical purposes, we can rightly say, as Bosanquet does, that *judgment* claims to be true, *i.e.*,

*presupposes the distinction between truth and falsity.* What then is a *proposition*? It should not be defined as the expression of a judgment in language, but rather as a *judgment expressed in language.* If we accept this definition, we can at once say that both judgment and proposition lay claim to truth, the only difference being that the proposition has linguistic reference, while judgment has not. We may also say that in logic we are concerned with propositions as understood, and a judgment is nothing but a proposition as understood. Thus instead of regarding judgment as merely subjective and proposition as merely objective, we regard both judgment and proposition as having both subjective and objective aspects, or as Johnson puts it, epistemic and constitutive aspects. We can reduce logic neither to psychology and metaphysics nor to grammar. If we decide to regard judgment and proposition as essentially the same, it will be possible for us to use the terms judgment and proposition in the same sense indifferently.

Subjective and objective aspects of judgment or proposition

What do we mean when we say that a judgment or proposition claims to be true? We have already seen that every proposition has two aspects, subjective and objective. Whenever we judge about something, and this something to which every judgment refers is its objective aspect. Every judgment has a subjective aspect as well, because judgment is a mental act, and involves belief or disbelief. This mental attitude is present in every judgment or proposition. To assert something is a mental process, and assertion is the essential mark of judgment. Consider the judgment 'gold is yellow'. It is a mental act, and I cannot say that gold is yellow without believing in the proposition. But if it be merely subjective, it cannot be either true or false. A mental state by itself, whether sensation, feeling or volition, cannot be regarded as either true or false. My idea of gold being yellow, if it is to be true, must have reference to reality or fact. That is, every judgment in order

to be true must correspond to some portion of reality. We may judge about the whole of reality or about some portion of it. According to Bradley, a judgment, in order to be true, must be compatible with some content of reality. Bosanquet also holds almost the same view, and according to him a judgment may have reference to some cross-section of reality or to the whole of it.

Judgment not always expressed in propositions.

We have defined proposition as judgment expressed in language, and have pointed out that since thought is the subject matter of logic, judgment and proposition are identical in essence. But should every judgment be expressed in language? This is not necessary, though every judgment *may* be so expressed with more or less difficulty. Where pictorial thinking is possible we can judge without the help of language, as in chess-playing. But we require language to think of complex matters such as the British constitution, the government of India, the respiration of plants, the internal structure of animals, etc., so as to give precision to our thought. In such cases we can hardly judge without the help of language. Though judgment and proposition are essentially identical, traditional logic gives us forms of proposition some of which hardly correspond to judgments, and in formal logic we cannot but consider these different forms of proposition. We shall discuss them in the next chapter.

Nature and function of the copula in proposition.

In discussing terms we pointed out that every proposition must have a *subject* and a *predicate* when fully developed. The proposition 'plants are living' has for its subject 'plants' and for its predicate 'living'. Here however we are considering the grammatical structure of a proposition, and seaking of the grammatical subject and predicate. The above proposition includes another word, 'are' which is called the *copula* of the proposition. It is commonly held that the

copula should always be some form of the verb 'to be' in the present tense.

What then is the *function* of the *copula*? According to Mill it is merely the *sign of predication*. 'Is' implies affirmation and 'is not' denial. In the proposition 'matter is extended' the predicate 'extended' is affirmed of matter, and the function of affirmation is performed by the copula 'is'. In the judgment 'some men are not happy' the predicate 'happy' is denied of the subject 'some men,' the copula 'are not' performing the function of denial. So the copula should *not* be regarded as a *third term* in a proposition. It is, according to Bosanquet, "the grip with which the parts of a single complex whole cohere with one another, differing according to the nature of the whole and the interdependence of its parts." According to Johnson, it is the tie that connects the subject and the predicate into a single whole. We have already remarked that every proposition is a continuous idea and is a single act. The copula implies the act of judging. Though from the grammatical point of view the copula appears to be the third member of a proposition, it is not really so, if we take into consideration the full significance of the proposition. Besides, we shall subsequently find that there may be propositions in which both the grammatical subject and the copula are absent. Predication is the main function of a proposition.

Another point has to be cleared up. Does the copula 'is' imply existence? It does not. In the proposition 'man is mortal,' it is held that 'is' is not merely a sign of predication but it also implies existence at the same time. But this is not true. We judge about fictitious objects; e.g., 'the centaur', 'fairy', 'nymph', etc. In the proposition 'A golden mountain is a beautiful thing', the 'is' does not imply real existence. Though every proposition may be regarded as existential in some sense, strictly speaking the only existential judgments are those in which the term 'exist' occurs, e.g., 'matter exists'.

A distinction between *proposition* and *sentence* may be drawn here. The sentence is the unit of language, the proposition is the unit of thought. A sentence is either correct or incorrect, while a proposition is either true or false.

Sentence and proposition.

Though every proposition is a sentence, every sentence is not a proposition. Thus optatives, *e.g.*, 'Long live the King', imperatives, *e.g.*, 'Go home', and questions, *e.g.*, 'Are you coming?' are sentences but not propositions, as they do not claim to be true. We have said that in a fully expressed proposition both the grammatical subject and the copula are present, but there are judgments in which either the subject or the copula or both are absent. Exclamations may be regarded as the most elementary form of judgment, in which both the subject and the copula are absent. After looking at a thing we may say, 'Beautiful!' This exclamation is a judgment meaning that, 'the thing is beautiful'. But in the proposition 'Beautiful!' both the subject and the copula are absent. Similarly 'Lightning!' is a proposition implying 'there is lightning' or 'that is lightning'. In impersonal propositions, *e.g.*, 'it is raining', 'there is a British constitution that protects the liberty of the people', etc., the whole proposition is concentrated in the predicate, and practically speaking no grammatical subject is present. In such propositions as 'that is a house', 'this is a book', etc., which may be called demonstrative propositions, though the grammatical subject is present it is indefinite. Thus there is a regular hierarchy of propositions, so to say, representing the gradual unfoldment of thought or judgment, and this enables us to pass from exclamatory propositions to impersonal ones, and from these to demonstrative propositions, till we reach propositions in which the subject is definite and all the parts of a proposition are fully stated, *e.g.*, 'Socrates is mortal', 'man is mortal', etc. We may also point out here that in such a proposition as 'fire burns', the copula 'is' is not separated from the predicate. In the next chapter we shall discuss the forms of proposition, beginning with the traditional scheme.

## CHAPTER VII

## FORMS OF PROPOSITION

Various *classifications of proposition* have been suggested by logicians, no one of which can be regarded as complete by itself. The traditional scheme of classification, which is simple though incomplete, gives us a provisional basis for discussing the forms of propositions. *Traditional* logic classifies propositions according to *quality* into *affirmative* and *negative*; according to *quantity* into *universal* and *particular*; and according to *relation* into *categorical* and *conditional*, the last being further sub-divided into hypothetical and disjunctive. Besides the above classifications, propositions are distinguished according to meaning or import into verbal and real propositions. In a previous chapter we have explained the nature of verbal and real propositions. So we shall not explain this distinction again in this chapter. Further, logicians often make a distinction between simple and complex propositions. A simple proposition is a single proposition, e.g., 'Ram is honest', 'man is mortal', etc. A complex proposition is made up of more than one proposition, *e.g.*, 'Ram and Shyam are students, or 'Ram is laughing and Shyam is talking'. In the former case the complex proposition consists of the two propositions, 'Ram is a student' and 'Shyam is a student', and in the latter case the two propositions 'Ram is laughing', and 'Shyam is talking' have been combined by 'and'. A complex proposition may be made up of more than two propositions.

Traditional and Kantian classifications.

We shall provide at the end of this chapter a scheme of classification which is accepted in a general way by modern logicians, though our basis of discussion will be the traditional scheme of classification. We shall, in the next chapter, briefly discuss the

Hamiltonian scheme of classification by quantifying the predicate, and the equational theory of Jevons, which also involves the quantification of the predicate. Though Bosanquet has thrown much light upon the problem of the forms of proposition, and we shall have to improve the traditional scheme in the light of his discussion, yet his classification showing the development of judgments, though useful, is rather psychological than logical, and so it is not necessary for us to discuss his scheme.

In the previous chapter we have shown that, in certain judgments only the predicate is present, neither the subject nor the copula being expressed, and that in an impersonal judgment the implication of the judgment is to be found entirely in the predicate. But even such judgments can be stated fully in a form in which the subject, the copula and the predicate are all present. Thus the judgment 'house' may be translated as 'that is a house'. Similarly the judgment 'it rains!' may be developed into the proposition 'rain is falling'. So a judgment when fully expressed contains a subject, a predicate and the copula, and every judgment may, with more or less difficulty, be thus fully expressed. The proposition "fire burns" when fully expressed becomes "fire is burning". We may further observe that, though a proposition is a single act of assertion, yet the analysis of its linguistic form gives us the grammatical subject and the grammatical predicate, behind which the real subject is always present. In discussing the forms of proposition, we shall have to bring out the implications of the grammatical subject and predicate, as well as the relation that exists between them.

### The Fourfold Scheme of Classification according to Quality and Quantity.

Traditional logic gives us a fourfold classification of propositions. According to *quality* propositions, as we have seen, are either *affirmative* or *negative*. An *affirmative* proposition *affirms a predicate of a subject, e.g.*, 'cows are domestic animals' or 'John is an Englishman'. In a *negative* proposition the *predi-*

*cate is denied of the subject, e.g.,* 'men are not perfect', 'Socrates is not a Roman'. We may here note that, in a negative proposition the copula is 'is not', which is the sign of denial. Hobbes tries to reduce all propositions to affirmative ones by appending the negative sign 'not' to the predicate. According to him, the proposition 'men are not perfect' ought to be 'men are not-perfect', the expression 'not-perfect' being affirmed of the subject 'men'. But this is a misreading of the actual intent of the proposition. In this proposition what we actually do is to deny the predicate of the subject, and not to affirm anything of it, and this denial is performed by the copula 'are not'. So the negative particle 'not' ought to be appended to the copula 'are' as an integral part of it.

Besides affirmative and negative propositions, Kant gives us another distinction according to quality, *viz., infinite* propositions.

Distinction between negative and infinite propositions not tenable.

The form of an affirmative proposition is 'S is P', that of a negative proposition, 'S is not P', while the form of an infinite proposition is 'S is not-P'. Thus the proposition 'this flower is not-red' is an infinite proposition. But what does this proposition really mean? Is an infinite proposition distinct from a negative proposition? This cannot be established. 'This flower is not-red' means that this flower excludes red, and this can be expressed by the negative proposition 'this flower is not red'. If 'not-red' means nothing at all, the proposition becomes nonsense. If it means anything, it must mean 'something other than red'. If so, then the negative proposition 'this flower is not red' brings out the meaning quite clearly. So to distinguish infinite propositions from negative ones the 'not' merely introduces tautology and may become misleading. Later on however we shall find that for the sake of convenience formal logic often has recourse to the form not-S, not-P, etc. One other point may be mentioned. Logicians have regarded infinite propositions in the form 'S is not-P' as positive, while the proposition in the form 'S is not P' has been regarded as negative. This

distinction, though formally allowable, cannot be regarded as valid if the meaning of such a proposition is taken into account.

Universal and particular propositions.

According to *quantity*, propositions are distinguished as *universal* and *particular*. A *universal* proposition is one in which the predicate is affirmed or denied of the *entire denotation of the subject, e.g.*, 'All or every S is P', 'All triangles are rectilineal figures', 'No men are perfect', etc. A *particular* proposition is one in which the predicate is affirmed or denied of a *part of the denotation of the subject, e.g.*, 'Some S's are *or* are not P', 'Some men are not happy', 'Some flowers are red', etc. Certain *marks of quantity* are recognised by logicians. 'All,' 'every,' 'no,' 'each,' 'any,' etc., are marks of universality, while 'some,' 'a few,' 'not all,' 'most,' etc., are marks of particularity. A categorical proposition, which is one in which the predicate is unconditionally affirmed of the subject, has, according to traditional logic, four elements, *viz.*, the subject, the predicate, the copula, and the mark of quantity. Thus in the proposition 'All S is P,' S is the subject, P the predicate, 'is' the copula, and 'all' the mark of quantity.

The fourfold classification of propositions.

Now combining the distinctions according to quality and quantity we get the *fourfold scheme* of classification, *viz.*, Universal affirmative (A)—'All S is P;' Universal negative (E)—'No S is P;' Particular affirmative (I)—'Some S is P;' Particular negative (O)—'Some S is not P.' Thus the *symbols* A, E, I and O stand respectively for universal affirmative, universal negative, particular affirmative and particular negative propositions. For the sake of convenience we shall follow several other logicians, and use in this book the symbols *S a P* for universal affirmative, *S e P* for universal negative, *S i P* for particular affirmative, and *S o P* for particular negative propositions. These symbols not only give us the quality and quantity of a proposition but also indicate the subject and the pre-

dicate. Thus the symbol *S a P* gives us S as the subject, *P* as the predicate, and the sign *a* tells us that the proposition is universal and affirmative. Thus *S a P= All S is P; S e P=No S is P; S i p=Some S is P; S o P= Some S is not P.* Further, we shall use such symbols as *S′* for *not-S, P′* for *not-P, Q′* for *not-Q* and so on. Thus the proposition *S′ a P′=All not-S is not-P, S′ e P=No not-S is P, S i P′=Some S is not-P,* and so on.

A *term* is said to be *distributed* when it is *taken in its entire extent or denotation.* A *universal* proposition *distributes* its *subject* term, while a *negative* proposition *distributes* its *predicate* term. Thus universal affirmative propositions (A) distribute their subjects only, while universal negative propositions (E) distribute both their subjects and their predicates. Particular affirmative propositions (I) distribute neither the subject nor the predicate, while particular negative propositions (O) distribute their predicates only and not their subjects. Thus affirmative propositions do not ordinarily distribute their predicates and particular propositions do not usually distribute their subjects. The universal affirmative proposition *All S is P=All S is some P;* the universal negative proposition *No S is P=No S is any P*; the particular affirmative proposition *Some S is P=Some S is some P*; the particular negative proposition *Some S is not P=Some S is not any P.* When we say that all men are mortal, we mean that some things denoted by the term 'mortal' are identical with all things denoted by the term 'man'. Similarly the proposition 'no men are perfect' means that all things denoted by the term 'perfect' are outside all the things denoted by the term 'man'. In the same way the proposition 'some men are virtuous' means that some things denoted by the term 'virtuous' are identical with some things denoted by the term 'man.' The proposition 'some flowers are not red' implies that all things denoted by the term 'red' are other than some things denoted by the term 'flower.'

We may note that the theory of distribution of terms in a proposition rests upon the assumption that the subject and the

predicate of all propositions are read in their denotation or extension and not in their connotation or intension. We shall subsequently find that the denotative view of predication is not satisfactory in all cases.

The meaning of 'some'.

We may here note in passing that in particular propositions such as, 'Some men are honest', 'some flowers are red', etc. The 'some' does not exclude 'all'. When we say that some lions are tawny, we do not thereby deny the likelihood of all or most of the lions' being tawny. We simply leave the question open. But 'some' is inconsistent with 'none'. When we say that some flowers are red, we thereby deny the fact that no flowers are red. But we must not suppose from the above discussion that whenever 'some' is true 'all' is also true. From the proposition 'some men are honest' we cannot pass on to the proposition 'all men are honest'.

To sum up the theory of distribution, we may point out that, if only one term of an affirmative proposition is distributed, it is the subject term, and if only one term of a negative proposition is distributed, it is the predicate term. But an affirmative proposition may, in some cases, distribute its predicate as well, as in the proposition 'all equilateral triangles are equiangular,' because in this particular case both the subject and the predicate are co-extensive. A universal negative proposition also distributes both its subject and its predicate terms; *e.g.*, in the proposition 'No men are four-footed,' all men are excluded from all four-footed creatures.

## Classification of Propositions according to Relation

Categorical, hypothetical and disjunctive propositions defined.

According to *relation* propositions are either *categorical, hypothetical,* or *disjunctive.* The hypothetical and disjunctive forms are called *conditional.* A *categorical* proposition simply *affirms or denies a predicate of a subject,* that is, it makes an absolute or unconditional statement; *e.g.*, 'All S's are P',

'all virtuous men are trusted' or 'no birds are without wings'. "A *hypothetical* proposition is one in which the *predication* made in *one* proposition is asserted as a *consequence* from that expressed by *another*." (Welton). According to Joseph, "An hypothetical judgment *connects* a *consequent* with a *condition* which it does not however assert to be fulfilled." The forms of hypothetical judgments are—If P then Q; If S is P, M is N; If S is P, it is Q; or If S is P, P is Q. "A *disjunctive* judgment affirms *alternatives*" (Joseph), or asserts the truth of at least one of a number of alternatives; *e.g.*, S is either P or Q; S is either P or Q or M.

Existence of antecedent not necessarily affirmed.

A hypothetical proposition does not affirm the existence of its antecedent; e.g., 'If Hannibal had marched on Rome after Cannae, he would have conquered it;' 'if a body is given a certain movement and if no counteracting conditions are operative, it will continue for ever to move in the same direction and with the same velocity.' In these examples we do not affirm the existence of the antecedent, but only assert the relation of content, that is, we assert that if certain conditions are given, certain events will necessarily take place. So in a hypothetical proposition, according to Cook Wilson, the solution of one problem depends upon the solution of another. Though this is true, hypothetical judgments, being judgments claiming truth, must have reference to reality or existence. The proposition 'if Hannibal had marched on Rome after Cannae, he would have conquered it,' asserts that the condition of Rome was such that Hannibal would have conquered the city if he had marched upon it after Cannae, though he never did so.

Negative hypothetical propositions impossible, though formally recognised by logicians.

Can hypothetical propositions be negative? A hypothetical proposition is not negative simply because its antecedent or consequent is negative. Thus 'if S is not M, it is P,' 'if A is B, C is not D' are affirmative propositions. The view that whenever the consequent of a hypothetical proposition is negative, the proposition is negative, is

untenable because even in such a case a relation between the antecedent and the consequent is asserted, *e.g.*, if we daily walk for an hour in the morning we shall not suffer from dyspepsia. Indeed, the truth seems to be that a hypothetical proposition can never be negative, since its essential function is to assert the dependence of a certain consequent upon a certain condition, and if any dependence is denied, we cease to have a hypothetical proposition. But by logicians the form 'If S is M, it is not P' is sometimes, though less accurately, taken as a denial of the relation between the antecedent and the consequent, and regarded as a negative proposition.

Hypothetical propositions in their *denotative* form may become either universal or particular. When the antecedent has the universal sign, it is universal, and when it has the sign of a particular proposition, it is particular; *e.g.*, If any S is P, it is M; sometimes if S is P, it is M; always if S is P, it is M; In some cases if S is M, it is P., etc. Of the above examples the first and the third are universal and the second and the fourth are particular. A few concrete examples will illustrate the point: *e.g.*, 'Sometimes when men are worried, they commit suicide' (particular); 'always if a man is shot through the heart, he dies' (universal); whenever a man moves, be expends energy' (universal); 'sometimes if a man is ill, he cannot rise from his bed' (particular); etc. Though hypothetical judgment is abstract and universal, instances may occur in which, though there is a connection between P and M. M may not be the full ground of P, or may not be universally operative, or may be counteracted by other influences. In such a case we have the form 'If S is M, it may be P,' or 'if S is M, it need not be P.' These are examples of *modal particulars*. Hypothetical judgments in their perfect form are ideal or abstract, universal and necessary, but if they are expressed in a denotative form they become concrete and cease to have the perfection of hypothetical judgments.

Abstract universals can easily be transformed into hypothetical propositions. Thus the proposition, 'Right-angled triangles have the square on the hypotenuse equal to the sum of the squares on the other two sides' =' if a triangle is right-angled, the square on the hypotenuse is equal to the sum of the squares on the other two sides.' But in most cases a hypothetical proposition cannot be reduced to a categorical proposition, or vice versa, without distortion of meaning. Thus if the propositions 'gold is yellow', 'man is mortal,' etc., are translated into the hypothetical form, we get the two propositions, ' If gold is, it is yellow,' ' if man is, he is mortal', which suggest the possibility of the non-existence of gold and man. Similarly the proposition, ' If men are honest, they do not deceive, may be translated into the categorical form ' All cases of men's being honest are cases of men's not deceiving.' Here the abstract meaning of the hypothetical proposition disappears when it is transformed into the categorical form and thus becomes concrete.

Distortion of meaning when hypothetical propositions are reduced to categorical ones or vice versa.

We may now pass on to the consideration of *Disjunctive Propositions.* We have noted that a disjunction asserts *alternatives.* These alternatives may be either two or more in number. Thus, 'S is either P or Q,' ' S is either P or Q or R,' etc., are disjunctive propositions. *Disjunctions,* like hypothetical propositions, are *universal* and *necessary,* but every disjunction has a *categorical element* which hypothetical propositions lack. Thus disjunctive propositions are more concrete than hypothetical propositions. In a *perfect* disjunction the *alternatives* are *exclusive* and *exhaustive, e.g.,* 'This book is either historical or non-historical.' Here the subject must accept one of the alternatives, because they exclude each other and are also exhaustive at the

Nature of disjunction and its relation to hypothetical and categorical propositions.

same time, so that there cannot be any assertion outside them. Thus according to logicians such a proposition can be translated into four hypothetical propositions, *viz.*, 'If the book is not historical, it is non-historical; if it is not non-historical, it is historical; if it is historical, it is not non-historical; and if it is non-historical, it is not historical.' But we must note that the disjunctive proposition stated above is not equivalent to the four hypotheticals taken together. Just as hypothetical propositions go beyond categorical ones, so also disjunctive propositions go beyond hypothetical ones. Further, every disjunctive proposition has a categorical element in it, and is intermediate between a purely categorical proposition and a purely hypothetical one.

Examples of imperfect disjunction.

There are some disjunctive propositions, however, which do *not* give us *exclusive and exhaustive* alternatives. Such disjunctive forms are not perfect. In the proposition 'This flower is either white or red,' we have alternatives that are *exclusive* but *not exhaustive,* since a flower may be neither white nor red but green. In this case, if we affirm one alternative, we can deny the other, but not vice versa. We can say, 'If this flower is white, it is not red, and if it is red, it is not white,' but we cannot say, 'If it is not white, it is red,' because it may be green. Such a form of disjunction is based upon the principle of contradiction, the form of which is 'S is not both P and Q.' Thus we find that when the alternatives of a disjunctive proposition are contradictories, it may be reduced to four hypothetical propositions, but when they are contraries it can be reduced to two hypothetical propositions only. Again there are examples in which the alternatives are *exhaustive* but *not exclusive*. Having witnessed the striking successes of a certain man we may say that he is either intelligent or industrious. Here in denying one of the alternatives we affirm the other, but if we affirm one we do not necessarily deny the other. If the man

is not intelligent, he is industrious, and if he is not industrious, he is intelligent; but it is possible that he may be both intelligent and industrious. But if the alternatives be *neither* exclusive *nor* exhaustive, then there is *no disjunction* at all, as in that case no assertion is possible. Take the example, 'Every man is either honest or happy.' Here there is no disjunction, because a man may be neither honest nor happy, or he may be both honest and happy. So by denying one of the alternatives we cannot affirm the other, nor can we, by affirming one of the alternatives, deny the other.

Disjunctive propositions may be either universal or particular, but must always be affirmative.

*Disjunctions,* like hypothetical propositions, may be *either universal or particular.* Thus, 'Some S's are either P or Q'; 'Some nations are either dependent or free'; 'Some men are either happy or unhappy,' are examples of disjunctive propositions which are *particular.* Similarly the proposition, 'Every idle man is either incapable of work or morally blameworthy,' is a *universal* disjunctive proposition. Disjunctives also resemble hypotheticals in that in their denotative form they cease to be abstract, and become concrete. Though disjunction can be either universal or particular it must *always* be *affirmative.* If negatived, it ceases to be disjunction and becomes a compound proposition. 'S is neither P nor Q' is equivalent to the two simple propositions, 'S is not P' and 'S is not Q,' which, when brought together, come to be a single compound proposition. To take a concrete example, 'This man is neither happy nor virtuous' does not tell us what the man actually is, and there cannot be any assertion in this case. It is a compound proposition.

## Classification of Propositions according to Modality

The modal distinctions of propositions involve very difficult

Modal distinction of propositions according to Aristotle and Kant.

considerations, and the problem may appear puzzling to beginners. Yet it is desirable to give a brief account of it at this stage, without making the discussion very difficult. When students have acquainted themselves with the principles of induction, they will find the problem easy of solution. *Aristotle* provides us with a *fourfold division* of modal propositions. They are (1) *Necessary*—'S must be P,' (2) *Contingent*—'S is P,' (3) *Possible*—'S may be P,' (4) *Impossible*—'S cannot be P'. Scholastic logicians regard necessary propositions as A, contingent propositions as I, possible ones as O and impossible ones as E. Aristotle's standpoint is objective and rests upon the nature of the relation between the subject and the predicate. *Kant's threefold distinction* of propositions according to modality is generally accepted, but his view, being subjective, has rightly been criticised by different logicians. According to him a *necessary* proposition is universal and cannot even be reversed in thought. The form of such a proposition is 'S is P, necessarily' (or 'apodeictic'). An *assertoric* judgment is simply accepted for the time being, but may be thought of as otherwise; it may be expressed as 'S is P, actually.' A *problematic* judgment expresses a doubt in the act of assertion and may be expressed as 'S is P, possibly.' According to Kant, necessary, assertoric and problematic judgments represent different degrees of belief in the mind of the person judging. According to Sigwart, to say that a judgment is necessary is not the same as to say that it is necessary for a predicate to belong to a subject. The former standpoint is Kantian, the latter Aristotelian.

Modal distinctions explained.

Since judgments claim truth, every true judgment is necessarily always true. Therefore Kant's subjective distinction between necessary and assertoric judgments cannot be accepted. A necessary judgment is an assertoric one, but the assertion is more emphatic. Both necessary and assertoric judgments imply complete belief, as every judgment claims to be true. What then

is the distinction between necessary, assertoric and problematic judgments? First we may point out that *necessary and problematic* judgments are *conditional,* while the *assertoric* judgment is *categorical.* When the *condition* is *fully known,* we have a *necessary* judgment, which may be expressed in the form—'If S is P, it is Q;' 'If a triangle is equilateral, then its three angles are equal'. When the *condition* is not fully known, or when its operation is counteracted by external influences, we have a *problematic* judgment in the form 'If S is P, it may be Q'; 'if you read carefully, you may pass the examination'; 'if you take medicine, you may recover'. An *assertoric* judgment is a statement of some *fact of experience, e.g.,* 'This fruit is an orange', or 'all lions are tawny'. Another distinction which follows from the above is that necessary and problematic judgments involve inference, while an assertoric judgment simply records some fact of experience. If this be true, then a necessary judgment cannot be more certain than an assertoric judgment, because inferential knowledge depends upon experience.

From the objective point of view a necessary judgment states the operation of some *law,* .*g.,* 'Planets move in elliptical orbits.' It is necessary and universal because it holds good of all planets, known and unknown. It is the task of induction to establish such laws. An *assertoric* judgment is a statement of fact and not the expression of any law, *e.g.,* 'All the kings who ruled in France during the 18th century were named Louis', or 'This flower is red'. A *problematic* judgment makes an assertion *without the knowledge of a necessary relation* between the subject and the predicate, *e.g.,* 'A seedling rose may be produced different in colour from any roses with which we are at present acquainted.' This judgment implies that there is nothing inherent in roses (or in the laws regulating the production of roses) to render this impossible. So Joseph says that, "a problematic judgment is provoked by knowledge; it is problematic because of ignorance." But the above distinctions cannot be regarded as formal. Necessary and problematic judgments, it is apparent,

involve reflection, while assertoric judgments are independent of any such reflection. According to Welton, all particular judgments are problematic, all generic universal judgments and hypothetical judgments are apodeictic, all propositions based on mere uncontradicted experience are assertoric. Induction will throw much light upon the distinctions given here.

Ordinarily a necessary judgment is stated in the form 'S must be P,' 'all the angles of a triangle together must be equal to two right angles'. An assertoric proposition is expressed in the form 'S is P', 'some men are honest'. A problematic proposition is expressed in the form 'S may be P,' 'you may pass the examination.'

## Logical Form of Propositions

According to traditional logic all propositions must be expressed in the logical form so as to determine their character. According to traditional logicians, a logical proposition should have the sign of Quantity, the subject, the predicate, and the copula, which must be some form of the verb 'to be' in the present tense. We may now consider how different propositions are to be reduced to the logical form in accordance with the view of traditional. logic.

**Indefinite or indesignate propositions.**

Certain propositions are called *indefinite,* though it is better to follow Hamilton in calling them *indesignate,* since we use the term 'indefinite' to indicate propositions which lack the sign of Quantity. When they are to be transformed into logical propositions, the sign of Quantity is to be provided in conformity with their significance. Thus 'heat is a mode of motion,' when reduced to the logical form, becomes 'every heat is a mode of motion'. Similarly 'bodies have weight'=' all bodies are things that have weights'. The proposition 'trains run at regular intervals'='some trains are those that run at regular intervals'. The proposition 'birds are singing'='some birds are singing.'

Such propositions as '*most* S's are P', 'most Indians are

Hindus', '*few* S's are P', 'few men are geniuses', etc., are called plurative propositions. Plurative propositions are particular because they are indefinite. 'Most' does not exclude 'all'. 'Most Indians are Hindus' means that 'some (more than half) Indians are Hindus'. 'Few' has a negative force. 'Few' is consistent with 'none' but excludes 'all'. 'Few men are geniuses' is equivalent to '*most* men are not geniuses'. Therefore it is an O proposition. Similarly 'Few S's are not P' = 'most S's are P', and is therefore an I proposition. But 'a few' means 'some' and has no negative meaning. 'A few men are honest' = 'some men are honest'. Though plurative propositions are particular, one distinction between some plurative and some particular propositions should be noted. From two plurative propositions a conclusion may be drawn. But from two particular propositions no conclusion follows. Thus from the propositions 'most S's are M', 'most S's are Q' we may deduce the conclusion 'some Q's are M'. But from the propositions 'Some S's are M', 'some S's are Q' we can draw no conclusion.

Plurative propositions.

*Numerically definite propositions* are those in which something is predicated of some definite proportion of a class, *e.g.*, 'two-thirds of S's are P', 'sixty per cent of S's are P', 'three-fourths of the members of the Bengal Assembly are intelligent men', etc. Plurative propositions and numerically definite propositions are exponibles. Exponible propositions are those, the full meaning of which can be brought out by two propositions. Thus 'most Europeans are Christians' means that 'some (more than half) Europeans are Christians' and also that 'a few Europeans are not Christians'. Similarly when we say 'two-thirds of S's are P' we at the same time mean that 'one-third of S's are not P'.

Numerically definite propositions.

Exponible propositions.

In some propositions *predication is limited* by the consideration of time or some condition. In such cases we have examples of ***multiple quantification***,

Multiple quantification.

e.g., 'all men are sometimes happy', 'in some countries all foreigners are unpopular'. These are examples of secondary quantification. We may have examples of triple quantification as well, *e.g.*, 'in all countries all foreigners are sometimes unpopular'. Such propositions may be either universal or particular according to their meaning.

**Propositions secundi adjacentis, and tertii adjacentis**

Propositions are distinguished according as they are propositions *secundi adjacentis* or propositions *tertii adjacentis*. In the former the copula is not separated from the predicate, *e.g.*, 'the man runs' when reduced to the logical form becomes 'the man is running.' Similarly the proposition 'all those who love virtue love theological disputation' when reduced to the logical form becomes 'all lovers of virtue are lovers of theological disputation'. In proposition tertii adjacentis the subject, the copula and the predicate are fully stated, *e.g.*, 'no men are perfect', 'all lovers of virtue are lovers of theological disputation.'

**Exceptive propositions.**

The quantity of the subject of a proposition may be limited by exception. If this exception is definitely stated, the proposition is universal, *e.g.*, 'all but four students of the class are present'. But if the exception is left indefinite the proposition is particular, e.g., 'all the students of the class but a few are present' which is equivalent to the proposition 'some students of the class are present'. A numerically indefinite proposition is also particular, *e.g.*, 'some two hundred men are present in the meeting'.

When a compound proposition is to be reduced to the logical form, the constituent propositions are to be separately stated in the logical form. Thus the proposition 'Ram and Shyam are honest' is equivalent to the two propositions 'Ram is honest', and 'Shyam is honest'.

We may now show by some important examples how propositions are to be reduced to the logical form.

Some examples to explain how propositions are to be transformed into the logical form.

The following propositions are to be reduced to the logical form in the same way. The propositions 'none but the virtuous are happy', 'the virtuous alone are happy', 'only the virtuous are happy' when reduced to the logical form are equivalent to either of the following two propositions 'all happy persons are virtuous', 'no non-virtuous persons are happy'. 'A few Indians are Christians'='some (less than half) Indians are Christians'. 'Most men are poor'='some men (more than half) are poor'. 'Many men have assembled'= 'some men are those that have assembled'. 'All the persons who came here were satisfied'='all the persons who came here are the persons who were satisfied'. 'He ran twenty miles'= 'he is a person who ran twenty miles'. 'Some men walk in the street'='some men are walking in the street'. 'It rains'='rain is falling'. 'All men are not happy'='Some men are not happy'. 'Not all men are intelligent'='some men are intelligent'. 'Sweet are the uses of adversity'='all the uses of adversity are sweet'.

The students are to remember that when a proposition is to be reduced to its logical form its meaning is to be taken into consideration and in no case the meaning of the proposition is to be distorted. Words however may be altered without affecting the meaning.

## New Classification of Propositions.

New classification of propositions to remedy defects of the traditional scheme.

The traditional classification of propositions gives us forms which overlap and are not exhaustive. It supposes that every proposition must have a subject, a copula, and a predicate. which, we have found, is not always required by thought. Thus the traditional classification is not merely inexhaustive but also confusing. Modern logicians, including Russell, Johnson, Welton, Monahan and others, provide a new scheme of classification to remedy

the shortcomings of the traditional one. This new scheme provides us with three main classes, *viz.*, *simple*, *complex* and *general*.

1. *Simple* propositions are: (a) Subject—attribute, *e.g.*, 'This mango is sweet,' 'this house is beautiful,' etc. These are *perceptual* propositions, attributing some character to some definite object. (b) *Relational* propositions, may be (i) two-termed, three-termed, or multi-termed, e.g., 'John met James,' 'Othello killed Desdemona,' 'my house is between a garden and a river,' etc., or (ii) class propositions, *e.g.*, 'Roses are included in the class of flowers,' or 'the Ganges is a river,' etc. These propositions may have negative forms.

2. *Complex* propositions are combinations of propositions. They have four forms: (a) *Conjunctive* propositions, *e.g.*, 'James and John are coming', 'the man got up and ran to see the result of the game,' etc. A conjunctive proposition contains two or more propositions combined by 'and' (b) *Implicative* propositions, e.g., 'If you come, I shall be happy'; 'if you run, you may fall down', etc. Here the propositions are combined into a complex one through the relation of implication. (c) *Disjunctive* propositions contain two or more propositions, all of which cannot be true at the same time. The form of such propositions is 'S is not both P and Q'; 'Gandhi cannot be both an Indian and a German,' 'a definite portion of a river cannot be both wide and narrow', etc. (d) *Alternative* propositions are a combination of two or more propositions, one of which must be true, e.g., 'This man is either an Indian or a non-Indian,' or 'this flower is either sweet-scented or devoid of sweet scent,' etc. Some complex propositions may also have a negative form.

3. General propositions may be either (a) about *a whole class, e.g.*, 'All men are mortal,' 'all birds have wings,' etc.; or (b) about *some of a class,*—'Some scholars are unsocial,' 'some books are interesting', etc. General propositions may also have negative forms. We have already noted that 'some' does not exclude 'all', though it does exclude 'none'. 'Some' means 'at

least one' but it is compatible with 'all'. Some logicians wrongly suppose that 'some' excludes both 'all' and 'none.'

The following chart, provided by Welton and Monahan, illustrates the above distinctions:—

Propositions

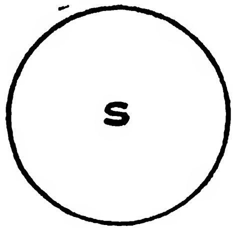
- (1) Simple
  - (a) Subject—attribute
  - (b) Relational
    - (i) Two-termed, etc.
    - (ii) Class propositions
- (2) Complex
  - (a) Conjunctive
  - (b) Implicative
  - (c) Disjunctive
  - (d) Alternative
- (3) General
  - (a) All
  - (b) Some

## CHAPTER VIII

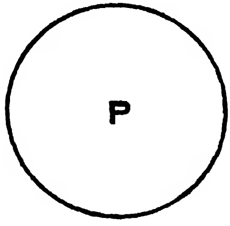
## DIAGRAMMATIC REPRESENTATION OF PROPOSITIONS

It is often useful to represent the *relation* between the *subject* and *predicate* of a proposition by means of geometrical *diagrams.* We give below the scheme of diagrams devised for this purpose by the Swiss logician, Euler. These are usually called after him *Euler's diagrams.*

The fundamental principle of these diagrams is that *all* the *individuals included* in any *class* or *denoted* by a *name,* are to be represented by a circle. Thus 'all the individuals included in the class S' (or 'all the individuals denoted by S') are represented by



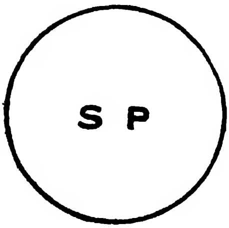

and 'all the individuals included in the class P' (or 'all the individuals denoted by P') are represented by



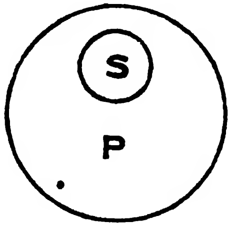

The symbols S and P, it is well to note here, are not arbitrarily chosen. They stand respectively for Subject and Predicate.

Then all possible relations between the class S and the class P are represented by the following *five* diagrams:—

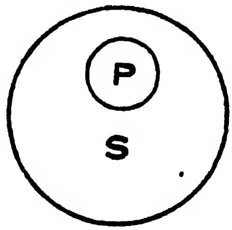
(i)



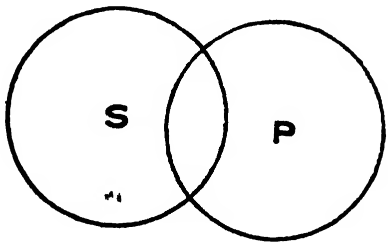

(ii)



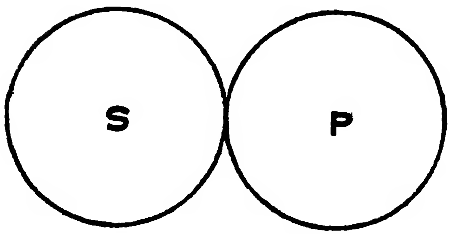

(iii)




(iv)




(v)




In diagram (i), the two circles *coincide*; *i.e.*, the class S and the class P are *co-extensive.*

Diagram (ii) indicates that *all* members of the class S are *included* in the larger class P.

Diagram (iii) indicates that all members of the class P are *included* in the larger class S.

In diagram (iv) the two circles *overlap,* indicating that *some* (but not all) members of the class S are *identical* with *some* (but not all) members of the class P.

In diagram (v) the two circles are completely *outside each other,* indicating that the two classes S and P have *no members in common.*

Let us see how these *diagrams apply* to the *fourfold scheme* of propositions:

(i) Where the two classes S and P are co-extensive, both the propositions, *viz.,* 'all S is (all) P' and 'some S is (some) P' are obviously true, that is to say, the diagram (i) *represents certain A propositions and certain I propositions. e.g.,* (A) all equilateral triangles are equiangular triangles; (I) some members of the Cabinet have divulged a Cabinet secret.

(ii) Where all the members of the class S are included in the class P. the propositions 'all S is (some) P' and 'some S is (some) P' are true, that is to say, diagram (ii) also *represents certain A propositions and certain I propositions. e.g.,* (A) All men are mortal; (I) some triangles are rectilineal figures.

(iii) Where all the members of class P are included in class S, both the propositions 'some S is P' and 'some S is not P' are obviously true, that is to say. the diagram (iii) *represents certain I propositions and certain O propositions. e.g.,* (I) Some ships are steamships; (O) some Hindus are not orthodox Hindus.

(iv) Where the classes S and P overlap, both the propositions, 'some S is P' and 'some S is not P' are obviously true, that is to say, the diagram (iv) also *represents certain I propositions and certain O propositions. e.g.,* (I) Some knaves are fools; (O) Some students are not diligent.

(v) Where the classes S and P have no members in common, both the propositions, 'some S is not P' and 'no S is P' are obviously true. that is to say, the diagram (v) *represents not only*

*certain O propositions but also all E propositions. e.g.,* (O) Some effects are not without a cause; (E) no human being is infallible.

Conversely,

An *A* proposition is always represented by
*either* diagram (i) *or* diagram (ii);

An *I* proposition is always represented by
*either* (i) *or* (ii) *or* (iii) *or* (iv);

An *E* proposition is always represented by diagram (v);

An *O* proposition is always represented by
*either* (iii) *or* (iv) *or* (v).

# CHAPTER IX

## IMPORT OF PROPOSITION

### Theories of Predication

In this chapter we are concerned with the import of categorical propositions only, and, with this object in view, shall examine the various theories of predication which are usually recognised. Without a clear understanding of the nature of predication the treatment of logic becomes impossible. Elsewhere we have thrown out some hints as to the import of propositions in discussing the nature of propositions. Before setting forth the correct view of predication, that is, what the import of propositions really is, we shall give an account of the five different theories recognised by logicians, *viz.*, (1) the Predicative view, (2) the Denotative view or the view of Class-inclusion, (3) the Comprehensive view, (4) the Connotative view or the view of Concomitance, and (5) the Indicative view. These are all answers to the problem, what really *the relation between the subject and predicate of a proposition is.* We may now examine these theories one after another.

The predicative view of propositions is natural and from the psychological point of view most satisfactory.

1. *The Predicative view.*—According to the predicative view, the *subject* of a proposition is to be read in its *denotation,* the *predicate* in its *connotation,* and the relation between them ought to be regarded as one of *possession* or *non-possession,* that is to say, every proposition. according to this view, is to be taken to mean that the thing or things denoted by the subject either *possess* or *do not possess* the *attribute* or attributes connoted by the predicate. Thus the proposition 'man is mortal' implies that human beings possess the attribute of mortality. 'No gold is white' means that the thing denoted by the term 'gold' does not possess the attribute of whiteness. From the psychological point of view, this theory

ordinarily appears to be the most satisfactory, as it seems to represent the natural trend of thought. When we say that 'all diamonds are combustible', we naturally think of the subject in its denotation, though it connotes attributes as well, and of the predicate in its connotation, though, in addition, it denotes things. In favour of this view it may also be pointed out that, we quantify denotation and not connotation, and the fact that it is the subject-term only which is quantified according to the traditional scheme also supports this predicative view. The quantification of the subject-term implies that it is to be read in denotation, whereas the fact that the predicate is not quantified suggests that it should be read in connotation.

Not all propositions naturally lend themselves to this interpretation.

But though the predicate of a proposition is not usually read in denotation, yet in some cases the class reference of the predicate is undeniable, as in the propositions, 'all owls are birds', 'all palms are endogens', 'all men are animals', etc. Further there may be propositions, though rare, in which it is natural to take the subject in connotation and the predicate in denotation, as in the proposition 'no plants with opposite leaves are orchids.' Thus it cannot be established that the predicative way of interpreting propositions is natural in every case.

Denotative view explained.

(2) *The Denotative or Class-inclusion view*.—Formal logicians, from the time of Aristotle, have been forced, by the inevitable logic of their mode of treatment of Logic, to accept the denotative or class-inclusion or class-exclusion theory of predication. According to this view both the *subject* and the *predicate* of a proposition are to be read in *denotation,* and the things denoted by the subject are to be viewed as *included* in, or *excluded* from, those denoted by the predicate. Thus the *relation* between the *subject* and the *predicate* is one of *inclusion* or *exclusion.* The proposition 'all owls are birds' means that the class of things

denoted by the term 'owls' is included in the class of things denoted by the term 'birds'. Similarly, 'Hindus are Aryans' means that the objects denoted by the term 'Hindus' are included in the objects denoted by the term 'Aryans'. In some cases, however, the predicate is included in the subject, if they are both read in denotation, *e.g.*, 'Some Aryans are Hindus.' Here the denotation of the term 'Aryans' is wider than that of the term 'Hindus', and therefore the former includes the latter. But in the proposition 'no triangle is a circle', or 'no bird is featherless', the denotation of the subject excludes the denotation of the predicate. The denotative mode of interpretation requires that the *predicate* should be always taken *collectively*. When we say, 'all owls are birds' or 'all men are animals', we do not mean that every owl is any bird or every man is any animal. What we do really mean, in each of these cases, is that, if we take the predicate collectively, the class denoted by it will include the things denoted by the subject-term. It is supposed that this mode of predication is convenient for logical manipulation, and such processes as conversion become possible if both the subject and the predicate are read in denotation. The diagrammatic representation of propositions is based upon the denotative interpretation. Further it is held, as we shall afterwards find, that syllogistic inference is not possible if both the subject and the predicate are not read either in denotation or in connotation. Aristotle's famous 'dictum de omni et nullo' also rests upon the assumption that the subject and the predicate of every proposition are read in denotation.

**Remarks upon the class mode of predication.**

But this view of predication is neither natural nor ultimate. It ignores the fact that propositions assert a *relation of content*. It does not recognise that every judgment is a single act of thought and is a unity. Though some propositions, in which the subject and predicate are class terms, easily lend themselves to this mode of interpretation, most propositions cannot naturally be so interpreted. Thus such propositions as 'some violets are white', 'some dogs are savage', etc.,

require that the subject should be read naturally in denotation, but the predicate, in connotation. Further some propositions do not express the relation of inclusion, *e.g.*, 'equilateral triangles are equiangular', 'Hyde is Clarendon,' etc., because in these cases the subject and the predicate are co-extensive. Moreover the first of these examples really asserts a relation of content. Therefore the connotative mode of predication is supposed to be better than the denotative. Hamilton's propositional schedule by quantifying predicates, and Jevons's equational scheme, of both of which we shall give a short account later on, rest upon the denotative view of propositions.

Comprehensive view explained.

(3) *The Comprehensive* *view*.—Sir W. Hamilton held that "every judgment expresses not only a quantitative relation in extension or denotation between subject and predicate, but also a similar relation in comprehension" (Welton). The *copula* in the former case means 'is contained under' and in the latter case, '*comprehends*.' We may explain the matter by an example. The proposition, 'all Hindus are Aryans', if viewed in extension, means that the Hindus are contained under class Aryans. If viewed in intension, the proposition means that the complex notion 'Hindus' *comprehends* or contains the *attributes* common to the class 'Aryans'. We have already, in connection with the denotative view of predication, noted the difficulties which arise, if both the terms are read in extension. Here we may note the difficulties which arise if both the terms are read in intension. If by 'comprehension' we mean all the common attributes of a class, then the proposition given above, and other similar propositions, become analytic as the predicate simply states some of the attributes which are implied by the subject-term, and we have found that analytic propositions do not contribute much to the enhancement of knowledge. If by 'comprehension' we mean connotation, then it is not true that the subject comprehends the predicate, for the connotation of the subject-term, if we mean by it conventional intension, may

Its defects.

not be larger than that of the predicate, *e.g.*, 'all equilateral triangles are equiangular.' Further this view suggests that intension, like extension, can be quantitatively measured, which is an absurd view. Moreover this, like the denotative view, ignores the unity of judgment and fails to see that judgments usually assert a relation of content. This view has all the defects of the denotative view and others in addition.

Preliminary criticism of certain other views.

(4) *The Connotative view.*—Before stating his own view of predication, Mill criticises a theory common to Hobbes and some other thinkers, who hold that a proposition implies a relation between two ideas. This view, according to Mill, is indefensible. He rightly points out that 'fire causes heat' does not mean that the idea of fire causes the idea of heat. We have already pointed out that every judgment is a unity, and is a single idea relating to some aspect of reality or of the whole of it. Similarly Spencer's view that a proposition is a transition from one idea to another cannot be accepted. According to him, 'man is mortal' means that we pass from the idea of man to the idea of mortality or of mortals. But the essence of every judgment being assertion, we cannot rightly say that a judgment is nothing but a psychological transition. Mill rightly criticises Hobbes. According to Hobbes, every judgment involves a belief that the predicate is a name of that of which the subject is also a name. Thus according to him, 'man is mortal' means that 'mortal' is the name of that of which 'man' is also the name. Such an interpretation requires that the extension of the predicate should be equal to that of the subject. But this is not always true. 'Mortal' has a wider extension than 'man.' Such a mode of interpretation can be true only of verbal propositions, *e.g.*, 'Cicero is Tully.' Besides, it is connotation that determines denotation and not *vice versa*.

An explanation of the connotative view.

So Mill argues that both the *subject* and the *predicate* of a proposition, except in the case of some singular propositions, should be read in *connotation,* and the relation between them is to be viewed as one of *agreement* or *concomitance*. When the proposition is negative, the relation is one of disagreement. Thus the proposition 'man is mortal' means that whatever has the attributes connoted by the term 'man', has also the attributes connoted by the term 'mortal'. This view is better

than the comprehensive view, since it does not hold that the attributes connoted by the subject term comprehend or include the attributes connoted by the predicate term. It is better than the denotative view, inasmuch as it holds that since connotation determines denotation, agreement between the connotation of the subject and that of the predicate also implies agreement between the things denoted by the subject and those denoted by the predicate. This view may be better expressed in another way, *viz.*, that the *attributes* connoted by the *subject* are an *evidence* or a *mark* of the *attributes* connoted by the *predicate*. 'All S is P' means that the attributes connoted by S are in agreement with, or a mark of, the attributes connoted by P. 'No S is P' means that the attributes connoted by S are not in agreement with the attributes connoted by P. Similarly 'some S is P' means that the attributes connoted by S are sometimes not in agreement with the attributes connoted by P. According to Keynes this view is ultimate, because connotation really determines denotation. This view is not affected by the quantitative reading of propositions, since connotation implies denotation.

The Indicative view of predication is unnatural and almost useless.

(5) *The Indicative view.*—According to this view the *subject* is to be read in *connotation* and the *predicate* in *denotation*. This is the reverse of the predicative view. 'All S is P', according to this view, ought to mean that the attributes connoted by S indicate the presence of something belonging to the class P. Very few propositions can be naturally read in this way. But some instances may be given: 'No plants with opposite leaves are orchids', 'all that glitters is not gold' are cases in point. If we examine the second proposition we find that the subject here is attributive, while the predicate is a substantive. But this view of predication is most unnatural and does not serve any useful purpose. We cannot even logically manipulate propositions on the basis of this theory of predication. We should, therefore, summarily dismiss it.

*The Reasonable View of Predication*

The ultimate import of propositions explained.

We may point out that if existence is taken in a wide sense to include the world of mythology, fiction, mind, etc., it may be asserted with certainty that *every proposition asserts existence.* The terms of a proposition are names which stand either for things or for attributes, and they must exist either in thought or in some other sphere of the universe. So James says. "In the strict and ultimate sense of the word 'existence,' everything which can be thought of at all exists as some sort of object, whether mythical object, individual thinker's object, or object in outer space and for intelligence at large.' A universal affirmative proposition, *e.g.*, 'All S is P.' asserts that S and P belong to *the same sphere of existence* or universe of discourse. A universal negative proposition, *e.g.*, 'No S is P.' asserts that S and P do not belong to the same sphere of existence. A particular proposition, whether affirmative or negative (*e.g.*, 'Some S is P' or 'Some S is not P') asserts that S and P may or may not belong to the same sphere of existence. Besides, if we look at the proposition as a whole and not at its constituent terms, we also find that since every proposition is either true or false, it must assert either some aspect of reality or the whole of it. Every judgment asserts a relation of content, and must be consistent with the contents of reality in order to be true. Every proposition therefore is adjectival in nature, and is a single act of thought which is predicated of existence or the real world, which is substantial in character. In perceptual propositions the reference to reality is direct, *e.g.*, 'This flower is red.' In the case of such categorical judgments as are abstract and universal, *e.g.*, 'All triangles have their three angles equal to two right angles,' and hypothetical propositions, *e.g.*, 'If a triangle is right-angled then the square on its hypotenuse is equal to the sum of the squares on the other two sides,' the reference to reality is indirect. We may however point out that for convenience of logical manipulation we may break up the unity of a proposition into its constituent terms, and read them either in extension or in intension.

## The Existential Scheme of Classification.

Our treatment of propositions has shown that every proposition has reference to existence. Some logicians accordingly have given us an *existential scheme* of propositions. We may briefly point out its leading features. We have found that the four main theories of predication are: (1) the predicative, (2) the denotative, (3) the connotative, and (4) the indicative theory. The comprehensive theory is, in a way, an amalgam of the denotative and connotative views. Every affirmative proposition and every universal negative proposition, if reduced to the existential mode

of interpretation, yield four different types of proposition in accordance with the four different theories of predication. Thus the universal negative proposition, 'No S is P', may be reduced to the following four existential propositions: (1) 'There is no individual belonging to the class S which possesses the attributes connoted by P' (from the standpoint of the predicative view); (2) 'There is no individual common to the two classes S and P' (from the standpoint of the class-inclusion theory); (3) 'The attributes connoted by S and P respectively are never found conjoined' (from the standpoint of the connotative view); (4) 'There is no individual possessing the attributes connoted by S which belongs to the class P' (from the standpoint of the indicative theory). Similarly we may illustrate the existential scheme of propositions by taking an I proposition. The proposition 'Some S is P' may be reduced to the following four forms: (1) 'There are individuals belonging to the class S and possessing the attributes connoted by P' (predicative view); (2) 'There are individuals common to the two classes S and P' (denotative view) (3) 'The attributes connoted by S and P respectively are sometimes found conjoined' (connotative view); (4) 'There are individuals possessing the attributes connoted by S which belong to the class P' (indicative view). The existential scheme may be illustrated by other forms of propositions. But for our purposes the above is sufficient.

We may however point out that there is no need to adopt such a scheme of propositions. If we remember that every proposition has reference to existence, that suffices for the understanding of the import of propositions. Multiplication of propositional schedules only serves to make the study of logic confusing.

## Hamilton's Eightfold Scheme of Propositions.

According to Hamilton, it is the fundamental postulate of logic that whatever is implicit in thought should be explicitly expressed in language.

Hamilton's eightfold scheme of proposition illustrated.

Archbishop Thomson, following Hamilton, holds that in thought we quantify not merely the subject of a proposition but also the predicate. So like Hamilton he quantifies the predicate and gives us eight forms of proposition in place of the four traditional forms. They are: (1) All S is all P (U); (2) All S is some P (A); (3) Some S is all P (Y); (4) Some S is some P (I); (5) No S is some P ($\mu$); (6) No S is any P (E); (7) Some S is not some P ($\omega$); (8) Some S is not any P (O). He expresses these eight forms of proposition by the following symbols:—SUP, SAP, SYP, SIP, S$\mu$P, SEP, S$\omega$P, SOP. We need hardly mention that Hamilton's classification of propositions rests upon the denotative theory of predication, because he reads both the subject and the predicate in denotation.

Some remarks.

Though Hamilton claims special merit for his scheme as he thinks that by means of it propositions can be reduced to equations and all propositions can be simply converted and further syllogistic reasoning becomes easy, yet it seems to us that this Hamiltonian scheme introduces unnecessary complications. Even if we accept the eightfold scheme, all propositions cannot be reduced to the equational form. Besides, his eight forms can be reduced to five. So it seems to us that the traditional four-fold scheme is sufficient for the purpose of logic. (Students may consult Keynes's book on Formal Logic for details).

**Brief Account of Jevons's Equational Schedule of Propositions.**

The three kinds of logical equation.

Jevons attempts to reduce all propositions to logical or mathematical equations. The equational scheme of propositions, as we have already found, rests upon the denotative theory of predication. Equations may be of three kinds : (1) *Simple*, (2) *Partial*, (3) *Limited*. The following is an example of *Simple Equation* : 'All exogens are dicotyledons' is equivalent to 'Exogens = Dicotyledons'. The symbolic form is $S = P$, which may be reduced to the two propositions 'All S is P,' 'All P is S'.

*Partial Identity.*—'All men are mortal' is equivalent to 'Men = Mortal men'. The symbolic form is $S = SP$. Thus from 'All S is P' we may have two equational propositions, $S = SP$, $SP = S$.

*Limited Identity.*—This form is not distinct from simple identity. The following is an example : 'All equilateral triangles are equiangular', which may be equationally expressed as 'Equilateral triangles = Equiangular triangles. The symbolic form is $VS = VP$, which means that within the class V, $S = P$.

We may now see how A, E, I and O propositions can be expressed mathematically :

Reduction of propositions to equational forms.

'All S is P' may be expressed by $S = SP$ or by $SP' = 0$ ($P'$ standing for not-P).

'Some S is not P' may be expressed by $S > SP$ or by $SP' > 0$.

'No S is P' may be expressed by $SP = 0$ or by $S = SP'$. Similarly 'Some S is P' may be expressed by $SP > 0$ or by $S > SP'$.

Some remarks.

The view that a proposition is equivalent to a logical equation is psychologically indefensible. It cannot be grasped easily. We should not break away from the traditional schedule of propositions more than is necessary, though the equational scheme of propositions is not unworkable. We have already remarked that the multiplication of propositional schedules leads to unnecessary difficulties. Logic is not mathematics and propositions need not be expressed by equations. (For details readers may consult Jevons.)

## CHAPTER X

## IMMEDIATE INFERENCE: OPPOSITION AND EDUCTION

### General Remarks on Inference.

*Inference* is a mental act by means of which the truth of a proposition is derived from the truth of other propositions. The passage from judgment to inference is not abrupt or sudden. In judgment we directly refer some idea which is clear to us to some content or element of reality. In inference also there is the same reference of content or reality, but here the reference to reality is not direct but is made through the medium of other judgments. Joseph defines inference as "a process of thought which, starting with one or more judgments, ends in another judgment whose truth is seen to be involved in that of the former." Every inference involves a movement of thought, since in every inference the truth of a proposition is derived from other propositions. The name 'inference' is given both to the process of thought and to the conclusion thus derived.

The process of thought involved in inference yields, as its result, the derived proposition, which is called the *conclusion.* The propositions from which the conclusion is derived are called the *premises* or the *data.* Thus in the inference 'No men are perfect, Socrates is a man, therefore Socrates is not perfect,' the first two propositions are the premises or data, while the third is the conclusion. Thus every inference requires some premise or premises which bear out the conclusion by way of implication. The implications of the data of inference are unfolded in the conclusion. Thus the conclusion necessarily follows from the premises. But though this is true, the conclusion must not be a repetition of the implications of the premises; it must impart

some new information, without which inference fails to contribute to the advancement of knowledge. Inference therefore requires that the *conclusion* should *follow* necessarily from the *premises* and at the same time *go beyond* them; that is, it must be in the premises and yet outside them. This is the paradox of inference. So Bosanquet defines inference as consisting in "asserting as fact or truth, on the ground of certain given facts or truths, something which is not included in those data." Though the premises are the ground of inference, they yield a new result as the consequence or conclusion.

Since inference is a mental act, it involves *construction*, inasmuch as the elements in the premises have to be synthesised or brought together to yield the conclusion. So according to Bradley inference involves synthesis and perception. Since inference brings out the nature of proof, just as proposition does that of assertion, there is *demonstration* in inference. The conclusion of an inference is *proved* by the premises, which are the evidence. Proof or demonstration therefore consists in seeing or perceiving that the synthesis or connection established in the conclusion is involved in or is implied by the premises. On this ground Bradley's remark that inference involves synthesis and perception holds good. Johnson compares inference with assertion. Assertion of a proposition is a mental act, so is inference. Inference is related to implication, according to Johnson, in the same way as assertion is related to proposition. He means that an inference unfolds the implication of premises just as an assertion unfolds the nature of a proposition. Inference, however, according to him, being a mental act, is distinct from the relation of implication, just as an assertion, which is a mental act, is distinct from a proposition, which is a statement about some fact. Since the validity of the conclusion of an inference follows from the given premises, every inference admits of formal treatment. In every inference the validity of the conclusion depends upon its being consistent with the premises. Formal logic requires only formal consistency, that is, it only demands that in inference the *conclusion* must be *consistent* with the *premises* and must follow from them. It does not enquire whether the premises are ultimately true or valid. But in the wider sense of validity, the validity of the conclusion depends as much upon the validity of the premises as upon its being consistent with them.

Different kinds of inference, *viz.*, Immediate inference, Syllogism and Inductive inference.

Inference may be either *immediate* or *mediate*. *Mediate* inference may be either *deductive*, that is, *formal*, or *inductive*, that is, *material*. In *immediate* inference the conclusion follows from *one premise* only, *e.g.*, 'All monkeys are animals (premise), therefore some animals are monkeys' (conclusion); 'all men are rational (premise), ∴ no men are irrational' (conclusion), etc. In such an inference the implication of one premise is unfolded in the conclusion. *Syllogism* or ratiocination is the name of the formal or *deductive mediate* inference. In syllogism a conclusion is drawn from *two* given premises, *e.g.*, 'All stones are hard, marble is a stone, therefore marble is hard'. In formal or deductive inference we do not enquire whether the premise or premises are true or false. To establish the validity of a certain piece of deductive inference it is sufficient to show that the conclusion legitimately follows from the given premise or premises. *Induction* is another form of *mediate* inference, in which a *general conclusion* is drawn from *observed instances, e.g.*, 'this cow is herbivorous, that cow is herbivorous, and all the cows observed are herbivorous without exception, therefore all cows are herbivorous'.

In *deductive* inference the *conclusion* can *never* be *more general* than the *premises*, but in *inductive* inference as stated above, the conclusion is *always more general* than the premises. In deduction we start with general premises, and establish a conclusion which must be either equally general or less general, but never more general, than the premises. In induction we start with observed particular instances, and from them pass on to a conclusion which is always more general than the data. The direction in which thought moves in deductive inference is the opposite of that in which it moves in deductive inference. Though this distinction is true, we shall find in the course of our discussion that deduction and induction are complementary, and neither can do without the other. We do not subscribe to the view that all inference, whether deductive or inductive, can be reduced to

syllogism or ratiocination, which according to some logicians is the only process of reasoning. We shall find as we proceed that though induction and deduction are allied to each other, they represent two distinct processes of thought. In this volume we shall discuss only the implications of deductive inference, and shall take up inductive inference in the second volume.

Immediate inference properly re garded as inference.

Some logicians hold that immediate inference is not inference proper. According to Mill, immediate inference is nothing but equipollency or equivalence of propositions. He means that immediate inference involves nothing but verbal transformation. Though it is true that in some cases of immediate inference the movement of thought is slight, yet since immediate inference unfolds the implication of a certain given premise, it can properly be regarded as inference. "The step from premise to conclusion in an immediate inference is small, but this does not prove that it is no step at all" (Welton). In this chapter we shall deal with immediate inference under the two main heads of *Opposition* and *Eduction.*

**Opposition of Propositions.**

Meaning of oppo sition.

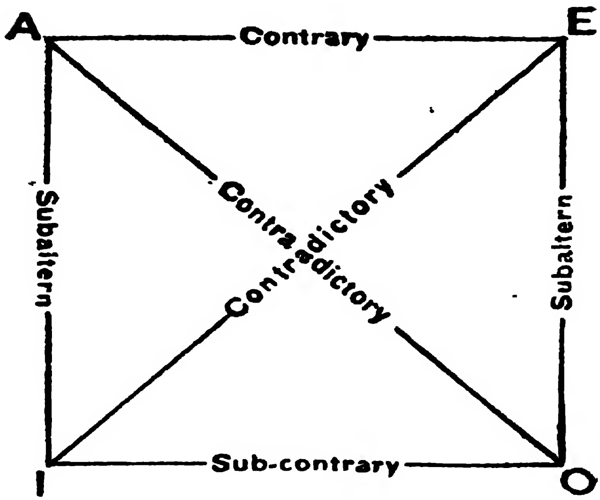
"*Two propositions* are technically said to be *opposed* to each other when they have the *same subject* and *predicate* respectively, but *differ* in *quantity* or *quality* or both" (Keynes). "Opposed propositions differ in form but refer to exactly the same matter, that is, to the same things, at the same time, and under the same circumstances" (Welton).

Four kinds of opposition.

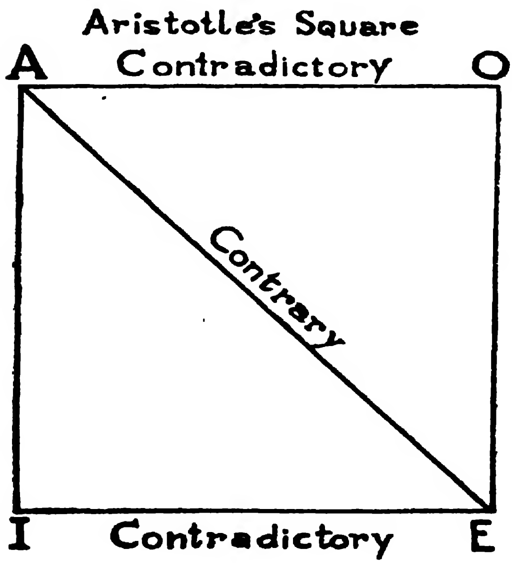
Four kinds of opposition of propositions are ordinarily recognised in logic, *viz.*, Contrary opposition, Sub-contrary opposition, Contradictory opposition and Subalternation. If two *universal* propositions having the same subject and predicate differ only in *quality*, we have *contrary opposition.* Thus the A proposition,

'All S is P', and the E proposition, 'No S is P', are opposed to each other, and their opposition is called contrary opposition. Both are universal propositions, and as such agree in respect of quantity; but while the one is affirmative, the other is negative, and thus the two differ in quality. If two *particular* propositions having the same subject and predicate differ only in *quality,* we have *sub-contrary opposition.* Thus the opposition between *I,* 'Some S is P', and 'O', 'Some S is not P', is sub-contrary opposition. We have *contradictory opposition,* when two propositions having the same subject and predicate differ both in *quality and quantity.* Thus the opposition between A, 'All S is P,' and O, 'Some S is not P,' as well as that between E, 'No S is P', and I, 'Some S is P', is contradictory opposition. We have the opposition of *sub-alternation* between two propositions having the same subject and predicate when they differ in *quantity* only but *not* in *quality.* Thus the opposition between A, 'All S is P', and I, 'Some S is P', and between E, 'No S is P', and O, 'Some S is not P', is subaltern opposition. In *subaltern* opposition the *universal* proposition is called the subalternant or *subalternans,* and the corresponding *particular* proposition is called *subalternate,* or *subaltern.* Thus an A proposition is the subalternant of its corresponding I proposition, and the I is the subalternate of the A. Similarly an E proposition is the subalternant of the corresponding O proposition, which is its subalternate or subaltern. These relationships are exhibited in the following diagram, known as the *Square of Opposition.* The meaning of the Square is obvious; so we need not dilate upon it. Aristotle however gives his own Square of Opposition, which differs in some important respects from the common square.

Aristotle does not recognise subalternation and sub-contrariety as forms of opposition. He recognises only contrary and contradictory oppositions.




*The Common Square of Opposition*




*Aristotle's Square of Opposition.*

"The doctrine of opposition may be regarded from two different points of view, namely, first as a relation between two given propositions; and, secondly, as a process of inference by which one proposition being given either as true or as false, the truth or falsity of certain other propositions may be determined" (Keynes). We have shown how the propositions A, E, I and O are opposed to each other. We may now point out how opposition involves inference.

Illustration of opposition as involving inference.

When two propositions are opposed to each other as *contrary* propositions, if *one* of them is *true*, the *other must be false*, but *both* the propositions *may be false* at the same time. If the proposition 'Everything in Aristotle is true' is true, then the proposition 'Nothing in Aristotle is true' is false. Similarly if the latter proposition, which is an E proposition, is true, then the former proposition, which is an A proposition, must be false. But it does not follow that if one of the above propositions is false the other must be true, for both the propositions 'Everything in Aristotle is true' and 'Nothing in Aristotle is true' may be false at the same time. Between two *sub-contrary* propositions the relation is such that if *one* of them is *false*, the *other must be true*, but *both may be true* at the same time. Thus if the proposition 'Some men are God-fearing' is false, then the proposition 'Some men are not God-fearing' must be true, and *vice versa*. But both the propositions, 'Some men are God-fearing' and 'Some men are not God-fearing' may be true at the same time. It will be apparent now that contrary opposition rests upon the principle of contradiction, which may be symbolically expressed as 'A is not both X and Y.' From this it follows that if A is X it is not Y, and if it is Y it is not X. But sub-contrary opposition rests upon the principle of excluded middle, which may be symbolically expressed as 'A is either X or Y', which means that if A is not X it is Y and if it is not Y it is X.

Oppositions as a Doctrine of Inference.

The relation between two contradictory propositions is such

that if *one* of them is *true*, the *other must be false*, and if *one* of them is *false*, the *other must be true*. Thus contradictory opposition rests upon the principles of contradiction and excluded middle at the same time. Thus if the A proposition, 'All Mussalmans are brave' be true, then the O proposition, 'Some Mussalmans are not brave', must be false. Again if the former proposition be false, then the latter must be true. Similarly if the E proposition, 'No philosophers are practical men' be true, then the I proposition, 'Some philosophers are practical men' must be false. In the same way if an E proposition is false, then the corresponding I proposition must be true.

Between two *subaltern* propositions the relation is such that if the *universal* proposition is *true*, then the corresponding *particular* is *also true*, but *not conversely*; that is, if the *particular* proposition be true, then the corresponding *universal* proposition *need not be true*. Again, of two subaltern propositions, if the *particular* be *false*, then the corresponding *universal must be false*, but not conversely; that is, if the *universal be false*, then the corresponding *particular need not be false*. Thus if the A proposition, 'All lovers of virtue are lovers of theological disputation' be true, then the corresponding I proposition, 'Some lovers of virtue are lovers of theological disputation' must also be true. But if the I proposition 'Some men are happy' be true, then the corresponding *A* proposition 'All men are happy' need not be true. Again if the proposition 'Some cows are carnivorous' be false, then the proposition 'All cows are carnivorous' must also be false. But if the proposition 'All men are happy' be false, then the proposition 'Some men are happy' need not be false. Similarly we may illustrate the relation between E and O propositions. Let us take the two propositions 'No graduates are illiterate' (E) and 'Some graduates are not illiterate' (O). It will be apparent that if the above E proposition be true, then the O proposition is also true. But if the proposition 'Some flowers are not red' (O) be true, the proposition 'No flowers are red' need not be true. Again if the proposition 'Some men are not two-footed' be false, then the

proposition 'No men are two-footed' must also be false. But if the proposition 'No men are musical' be false, then the proposition 'Some men are not musical' need not be false.

True opposition.

True opposition exists between propositions which cannot be true together, that is, which are incompatible. If we take opposition in this sense, then there are only two kinds of opposition, *viz.*, contrary opposition and contradictory opposition, that is, opposition between A and E and also between A and O and between E and I, as shown in Aristotle's Square of Opposition. In that case sub-contrary opposition and subaltern opposition cannot, strictly speaking, be regarded as opposition proper, because I and O may be true together, and in the same way A and I as well as E and O can be true at the same time. So Bosanquet, following Aristotle, regards sub-contrary opposition as no opposition at all. But in our discussion we have followed the usual practice of logicians. Though sub-contrary opposition and subalternation are not oppositions proper, yet a distinction exists between sub-contraries and subalterns which ought to be exhibited. Opposition has come to include all relations between propositions having the same subject and predicate but differing in form, whether such relations are incompatible or compatible. Though contrary and contradictory relations are the main types of opposition, yet since contradiction involves contrariety and also goes beyond it, it may be regarded as more effective than contrariety to disprove the truth of a proposition. Contrary propositions may admit of a mean between them, as the propositions 'All men are happy' and 'No men are happy' do, and therefore both may be false. But contradictory propositions allow no such mean and therefore one of them must be true and the other false, *e.g.*, 'All negroes are black' and 'some negroes are not black'. So *contrary* propositions *cannot both* be *true* at the same time but may both be false, but *contradictory* propositions can *neither both* be *true nor both* be *false* at the same time.

According to some logicians there are three types of opposition, namely, *Contrariety*, *Contradiction*, and *Sub-contrariety*.

Subalternation.

They hold that opposed propositions must differ in quality. But subaltern propositions differ only in quantity, but not in quality. But it seems that if subaltern propositions are not to be regarded as opposed to each other then sub-contrary propositions should also be similarly regarded because sub-alterns and sub-contraries are not incompatible and may be true at the same time, as we have already pointed out.

We may now provide a chart to show how the doctrine of opposition is a doctrine of immediate inference:

If A is true, E is false; I true, O false.
If E is true, A is false; I false, O true.
If I is true, A is doubtful, E false, O doubtful.
If O is true, A is false, E doubtful, I doubtful.
If A is false, E is doubtful, I doubtful, O true.
If E is false, A is doubtful, I true, O doubtful.
If I is false, A is false, E true, O true.
If O is false, A is true, E false, I true.

We mav now note in passing that between two *singular* propositions having the same subject and predicate but *differing* in *quality*, the only kind of opposition that can exist is *contradictory* opposition; *e.g.*, 'Socrates is honest' (A) and 'Socrates is not honest' (E). Such an opposition is called *secondary* opposition. In this case if we affirm one of the propositions we are bound to deny the other, and if we deny one we are bound to affirm the other, that is, if one is true the other is false and *vice versa*. Keynes points out that even in the case of singular propositions the ordinary squares of opposition can be obtained by quantifying the predicates. Thus of propositions in which 'Browning' is the subject and 'obscure' the predicate, we have the following four forms:—'Browning is always obscure' (A), 'Browning is never obscure' (E) (contrary), 'Browning is sometimes obscure' (I) (subalternate), 'Browning is sometimes not obscure' (O) (contradictory). But if a singular proposition is interpreted in different ways by quantifying the predicate, it ceases to be a singular proposition, because the

Opposition of singular propositions.

subject no longer remains a *whole indivisible*. Though this is true, such manipulation of singular propositions is not useless, because 'Browning is obscure' does mean that he is obscure at times but not always.

Opposition of propositions is mainly a formal distinction and not a material one, as has been clearly brought out by the previous discussion. We may also remark that since the truth of I follows from the truth of A and the truth of O from that of E, if the two contrary propositions A and E were true together, it would follow that the contradictories A and O as well as E and I became true together, which is absurd. We have previously held that contradictory opposition exists between A and O and also between E and I. But if 'Socrates is honest' and 'Socrates is not honest' are contradictories, then A and E may be contradictories at times. So we may say that two propositions having the same subject and predicate, whether they differ both in quality and quantity or only in quality, may be contradictories, if the positing of one involves the sublating of the other and the sublating of one involves the positing of the other. A complex proposition can be contradicted by another complex proposition. Thus the proposition 'He came here and saw me' can be contradicted by the complex proposition 'Either he did not come here or he did not see me'. Similarly if we take the meaning into consideration and not the form only, then two affirmative propositions can be contraries, *e.g.*, 'This flower is green' (A) and 'This flower is red' (A). If one of the propositions is true then the other is false, but both may be false, since the flower may be yellow. We have already remarked that a proposition is not denied by its contrary as effectively as by its contradictory.

Contradiction and contrariety further considered.

The doctrine of opposition has been extended by traditional logic to *hypothetical* propositions also. We have already stated that such propositions assert a relation of content and are abstract, and when they are expressed in the denotative form they deviate from the ideal, but still they may be so expressed. Let us first consider the abstract forms and then the concrete forms of *hypothetical* propositions, to find out how the doctrine of opposition has been applied to them. The hypothetical proposition may be expressed by the symbol 'If X then Y', or 'If S is M, it is P'. Then we have the following four propositions :—(A) If X then Y, or If S is M it is P; (E, contrary) If X then not Y, or if S is M it is not P ; (I, subaltern) If X then perhaps Y, or if S is M it may be P; (O, contradictory) If X then not necessarily Y, or If S is M it need not be P. We may exemplify

Hypothetical propositions reduced to the four traditional forms, that the doctrine of opposition may be applicable to them.

the distinction by a concrete example : If the stone is material it occupies space (A); if the stone is material it does not occupy space (E); if the stone is material it may occupy space (I); if the stone is material it need not occupy space (O). Let us now consider the hypothetical proposition when expressed in the denotative form :—

(A) If any S is M that S is always P;
(E) If any S is M that S is never P (contrary);
(I) If an S is M that S is sometimes P (subaltern);
(O) If an S is M that S is sometimes not P (contradictory).

A concrete example is : If any man is honest he is always trusted (A); if any man is honest he is never trusted (E); if some men are honest they are sometimes trusted (I); if some men are honest they are sometimes not trusted (O). Of the above, the I and the O propositions can also be expressed as 'If some men are honest they may be trusted' and 'If some men are honest they need not be trusted'.

We need not consider how the doctrine of opposition has been applied to disjunctive propositions by traditional logic.

## Eductions

General remarks on eduction or immediate inference.

We have already seen that opposition is a process of immediate inference, as the truth or falsity of one proposition is implied by, or follows from, the truth or falsity of other propositions having the same subject and predicate but differing either in quantity or in quality or in both. We have already remarked that in immediate inference the implication of one proposition is brought out by another proposition, that is, in it the conclusion follows from a single judgment. We may now, having discussed opposition, consider other kinds of immediate inference known as *Eductions*. "Eductions are those forms of immediate inference by which, from a given proposition, accepted as true, we educe other propositions *differing* from it in *subject*, in *predicate*, or in *both*, whose truth is implied by it" (Welton). Eductions are of four main varieties, *viz.*, (1) *Conversion*, (2) *Obversion*, (3) *Contraposition*, and (4) *Inversion*. But some material processes of immediate inference are also called educ-

tions. In considering eductions we are not concerned with tautologous propositions such as S is S, or with propositions which are self-contradictory, *e.g.*, S is not S. Every categorical proposition has two terms, the subject and the predicate, which are generally symbolised by S and P. These terms suggest as their negatives not-S and not-P, which we shall express by the symbols S′ and P′ respectively. So a categorical proposition suggests to our minds four terms, and in every such proposition two of the above terms must be present. Further, as we have already noted, we shall use the symbols S a P for All S is P, S e P for No S is P, S i P for Some S is P, and S o P for Some S is not P. The proposition P a S will imply All P is S, and so on. The proposition S′ a P′ will imply All not-S is not-P, and so on. With these observations we may now consider conversion, obversion, contraposition and inversion which are the main types of formal immediate inference.

## 1. Conversion

Nature of conversion.

"A proposition is *converted,* when its *subject* is *made* the *predicate,* and vice versa, its *quality* (affirmative or negative) remaining *unchanged*" (Joseph). Conversion thus requires the terms of the original proposition to be transposed in the conclusion: S, which is the subject of the given proposition, becomes the predicate of the conclusion, and P, which is the predicate of the given proposition, becomes the subject of the conclusion. The original proposition is called the *convertend* and the conclusion is called the *converse.* In converting a proposition we must observe the following two rules: (1) The *converse* must be the *same in quality* as the *convertend*; (2) No term must be *distributed* in the *converse* if it was not *distributed* in the *convertend.* (But a term distributed in the convertend may be either distributed or undistributed in the converse).

When converted, A becomes I. S a P when converted yields

P i S as the conclusion. From All S is P we get Some P is S by conversion. 'All Mussalmans are brave' (convertend), 'Some brave men are Mussalmans' (converse). In conversion the predicate term, like the subject term, is read in extension. Keynes says, "In the process of converting a proposition, however, the extensive force of the predicate is made prominent, and an import is given to the predicate similar to that of the subject." Thus if the predicate of a proposition is an adjective, it has to be reduced to a substantive before conversion. Thus by converting the proposition 'Socrates is wise' we get the conclusion 'A wise man is Socrates'. The predicate term 'wise' in the original proposition was an adjective, but as the subject of the converse it has become a substantive. Similarly 'All men are mortal', when converted becomes 'Some mortal things are men'. "All monkeys are animals', when converted, becomes 'Some animals are monkeys'. In this case the predicate term of the original proposition is a substantive and its denotative significance is apparent, and such a proposition can be converted with greater ease than a proposition which has an adjective as its predicate term. We should also remember that to educe one proposition from another we have to reduce the premise to its logical form. Thus the proposition 'A stitch in time saves nine' has to be reduced to its logical form, *viz.*, 'All stitches in time are things that save nine stitches', from which we may draw the conclusion 'Some things that save nine stitches are stitches in time'. By conversion we get an interpretative proposition as the conclusion.

Conversion of A propositions. Formally we convert A per accidens or by limitation. Sometimes knowledge of fact may enable us to convert A simply.

The above examples show that formally we cannot get an A proposition as the conclusion by converting A. If we passed to an A proposition by conversion from an A proposition, we must have transgressed a rule of conversion. Take the proposition 'All S is P'. The first rule requires that by converting it we must arrive at an affirmative proposition as our conclusion. In the

given proposition, P, which is the predicate of an affirmative proposition, is not distributed, and the second rule requires that since it is not distributed in the original proposition, it must not be distributed in the conclusion. In conversion the subject of the original proposition becomes the predicate of the conclusion and the predicate of the given proposition becomes the subject of the conclusion. Now P, which is undistributed in the given proposition and is its predicate, must be the subject of the conclusion when converted and must be undistributed there. The conclusion must also be affirmative, the original proposition being affirmative. So the legitimate conclusion becomes 'Some P is S'. Thus we find that by converting an A proposition we get an I proposition as the conclusion. So the conversion of A is said to be per accidens or by limitation. The conclusion and the premise in this case are not equivalent propositions. Conversion of A involves a loss in the fulness of the meaning of the original proposition. By converting 'All monkeys are animals' we get the conclusion 'Some animals are monkeys'. But by reconverting the conclusion we cannot get back to the original proposition. 'Some animals are monkeys', when converted, becomes 'Some monkeys are animals', which is not the same as the original proposition 'All monkeys are animals'. Though formally it is not possible to pass from A to A by conversion, yet in some cases, where the subject and the predicate are co-extensive terms, our knowledge of facts may enable us to do so. Thus the proposition 'All equilateral triangles are equiangular' yields the conclusion 'All equiangular triangles are equilateral'. But we cannot, even in this case, without knowledge of geometry, have an A proposition as the conclusion purely on formal grounds. Similarly 'Chatham is the elder Pitt' (A) yields the conclusion 'The elder Pitt is Chatham' (A). So also the proposition 'Demosthenes and Cicero are the greatest orators of antiquity' (A) yields the conclusion 'The greatest orators of antiquity are Demosthenes and Cicero'. "The conversion of propositions may be studied formally, with symbols for terms, but when real terms replace the symbols they must

affect the judgment, and our treatment of it in conversion." Formal treatment is sound within its limits, but we have seen in the course of our discussion that form and matter are not separable. What is form in one case is matter in another. We must be careful in converting an A proposition, because very often the conversion of A leads to fallacy. Granted that it is true that all idle men are poor, it is not true that all poor men are idle. Similarly we cannot pass from the proposition 'All unemployed men are unhappy' to the conclusion that 'All unhappy men are unemployed. So the conversion of A should, as a rule, be per accidens or by limitation, though in some cases our material knowledge may justify us in converting A simply, that is, passing to an A proposition from an A proposition by conversion. But if we adhere strictly to the formal rules of conversion we can never convert A simply; its conversion must in every case be by limitation.

E converts to E and therefore the conversion of E is simple. In this case the convertend and converse are equivalent propositions

E converts to E. From S e P we get P e S by conversion. No S is P (convertend), No P is S (converse). 'No cows are carnivorous' (premise), 'No carnivorous animals are cows' (conclusion). Simple conversion of E is possible, because an E proposition distributes both S and P and we are justified in distributing them in the conclusion. 'No S is P' means that 'All S's are other than all P's', and therefore we may say justly that 'All P's are other than all S's', that is, from 'No S is P' we can pass on to the conclusion 'No P is S'. From the proposition 'No God-fearing men are unhappy' we may pass on to the conclusion that 'No unhappy men are God-fearing'. Since the attribute of being unhappy and the attribute of being God-fearing are absolutely incompatible, that is, one is absolutely different from the other, every object denoted by one of the terms is different from every object denoted by the other term. In the case of an E proposition, the subject and the predicate terms being absolutely different, the relation between them is reciprocal.

Thus both the convertend and the converse in an E proposition are equivalent propositions, that is, just as we can pass from No S is P to No P is S, so also we can pass from No P is S to No S is P.

I converts to I. Here also conversion is simple. S i P when converted becomes P i S. Some S is P (convertend), Some P is S (converse). 'Some Hindus are God-fearing' (premise), therefore 'Some God-fearing men are Hindus'. In the case of an I proposition both the subject and the predicate are undistributed, and therefore when we pass from S i P to P i S, there is no transgression of the rules of conversion. Both P i S and S i P are affirmative propositions, and in both of them both P and S are undistributed. 'Some diamonds are black, therefore some black things are diamonds'. As in the conversion of E, so here the premise and the conclusion are equivalent propositions. Though this is true, the conversion of I does not always yield a happy result. From 'Some men are teachers' we may pass on to the conclusion 'Some teachers are men', but here the conclusion suggests that there may be teachers who are not men, which is false. Welton remarks that "when we speak of the simple conversion of I, we do not mean that 'some' denotes the same proportion of the total denotations of the subjects of both convertend and converse".

I converts to I Here also conversion is simple and the convertend and converse are equivalent propositions.

O cannot be converted. As it is necessary that if the convertend be negative, the conclusion must also be negative, if we convert S o P, we have to pass to a negative conclusion, *viz.*, P o S. If we convert 'Some S is not P', we have to establish either the conclusion 'Some P is not S' or the conclusion 'No P is S'. But this is not possible, for S, which as the subject of a particular proposition, is not distributed in the premise, becomes distributed in the conclusion as being the predicate of a negative

O cannot be converted. But knowledge of facts may enable us to pass to an O proposition from an O proposition.

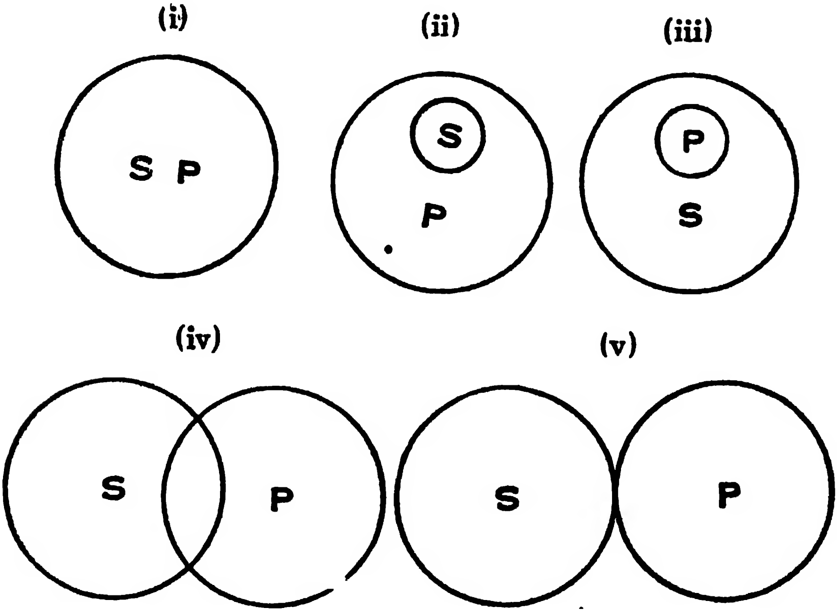
proposition; but this is forbidden by the second rule of conversion. Thus from the proposition 'some men are not wise' we cannot pass on to the conclusion 'some wise things are not men'. Similarly from the proposition 'some animals are not carnivorous' we cannot pass on to the conclusion that 'some carnivorous things are not animals'. So also from the proposition 'some men are not teachers' we cannot pass on to the conclusion 'some teachers are not men'.

Sometimes however material conversion of O is possible, just as sometimes the simple conversion of A is justified on the ground of material knowledge. Thus from the proposition 'some masons are not freethinkers' we can pass on to the conclusion 'some freethinkers are not masons'. Similarly we can pass from the proposition 'some men are not black' to the conclusion 'some black things are not men'. 'Some S is not P' means that the attributes connoted by some S are incompatible with the attributes connoted by all P. It does not mean that all P's are other than all S's. So Some S is not P and All P is S may be true at the same time. The propositions 'Some men are not wise' and 'All wise things are men' may be true together. To sum up the result from the formal point of view, A converts per accidens, E and I simply, and O not at all.

The rules for the conversion of A, I, and E propositions may be illustrated by Euler's diagrams (See page 137).

A, as we have previously seen, is represented by diagram (i) or (ii). In diagram (i) P and S are obviously co-extensive, *i.e.*, we know not only that all S is P but also that all (and therefore also some) P is S. But in diagram (ii) *only some* P is S. But there is nothing in the form of the proposition 'all S is P' to tell us whether it is represented by diagram (i) or diagram (ii); and that being so, all that we are justified in saying about P is what is common to the two cases, *viz.*, that *some* P is S., *i.e.*, A converts to I.

An I proposition may be represented by any of the first four diagrams. Here again, in diagram (i) and (iii) all, and therefore some, P is S, but in diagrams (ii) and (iv) *only some* P is S; and




once more, being given that some S is P, we are only justified in affirming of P what is common to all the four possible cases, *viz.*, that *some* P is S., *i.e.*, I converts to I.

E is represented always by diagram (v), from which it is plain that the classes S and P have nothing in common, and thus that *no* P is S., *i.e.*, E converts to E.

O is represented by diagrams (iii), (iv) and (v), but here we can say nothing about P that holds good of all three cases, and therefore O cannot be converted. For diagram (iii) shows *all* P as being S. diagram (v) *no* P as being S; and these two are inconsistent with each other.

## 2. Obversion

The general nature of obversion.

"*Obversion* is a process of immediate inference in which the *inferred proposition* (or *obverse*), whilst retaining the *original subject*, has for its predicate the *contradictory of the predicate* of the original proposition (or *obvertend*)" (Keynes). Obversion is also called Permutation, Equipollence, Infinitation, Immediate Inference by Privative Conception, or Contraversion. In obversion the original proposition or the premise is called the obvertend and the conclusion the obverse. The rule of obversion is, "*Negate* the *predicate* and *change* the *quality*, but leave the *quantity unaltered*" (Welton). By obverting A we get E: S a P (obvertend), S e P′ (obverse); All S is P (premise), No S is not-P (conclusion). From the proposition 'Barkis is willing' we get the conclusion 'Barkis is not not-willing' by obversion. 'All virtuous men are happy' (obvertend), 'No virtuous men are not-happy' (conclusion).

A obverts to E.

The obverse of E is A: S e P (obvertend), S a P′ (obverse). No S is P when obverted becomes All S is not-P. 'No men are perfect' (obvertend), 'All men are not-perfect' (obverse). When obverted, the proposition 'No triangles are four-sided' yields the conclusion 'All triangles are not-four-sided'.

E obverts to A.

I obverts to O. S i P becomes S o P′ when obverted. Some S is P (obvertend), Some S is not not-P (obverse). 'Some men are rich' (obvertend), 'Some men are not not-rich' (obverse).

I obverts to O.

O obverts to I. S o P becomes S i P′ when obverted. Some S is not P (obvertend), Some S is not-P (obverse). 'Some flowers are not red' (obvertend), 'Some flowers are not-red' (obverse).

O obverts to I.

We can, in obversion, replace a formal contradictory by a material contradictory, provided that there is no change in the meaning. Thus instead of saying that 'Barkis is not not-willing' we may say that 'Barkis is not unwilling'. Similarly we can replace the proposition 'No men are non-mortal' by the proposition 'No men are immortal'. But the proposition 'No virtuous men are not-happy' cannot be replaced by the proposition 'No virtuous men are unhappy', because 'unhappy' has not the same meaning as 'not-happy' and is not the contradictory of 'happy'. Similarly 'Some men are not-rich' cannot be replaced by the proposition 'Some men are poor', because 'poor' is not the contradictory of 'rich' and so it does not possess the same significance as 'not-rich'.

A formal contradictory may be replaced by a material contradictory if the meaning remains the same.

Since every positive proposition can be negatively expressed, Welton, like Mill, holds that obversion should not be regarded as a form of immediate inference, because it does not involve any movement of thought, the obverse being merely the verbal transformation of the obvertend. If this is conceded, then even conversion ceases to be a form of immediate inference, because it involves simply the transposition of terms. We may point out however that when an obvertend is given, it is not always clear what its obverse should be. Obversion also requires some mental activity, however slight. And in Logic it is not a question of the amount of mental activity expended, but a question of reasonableness involved in an act of passing from one proposition on to another.

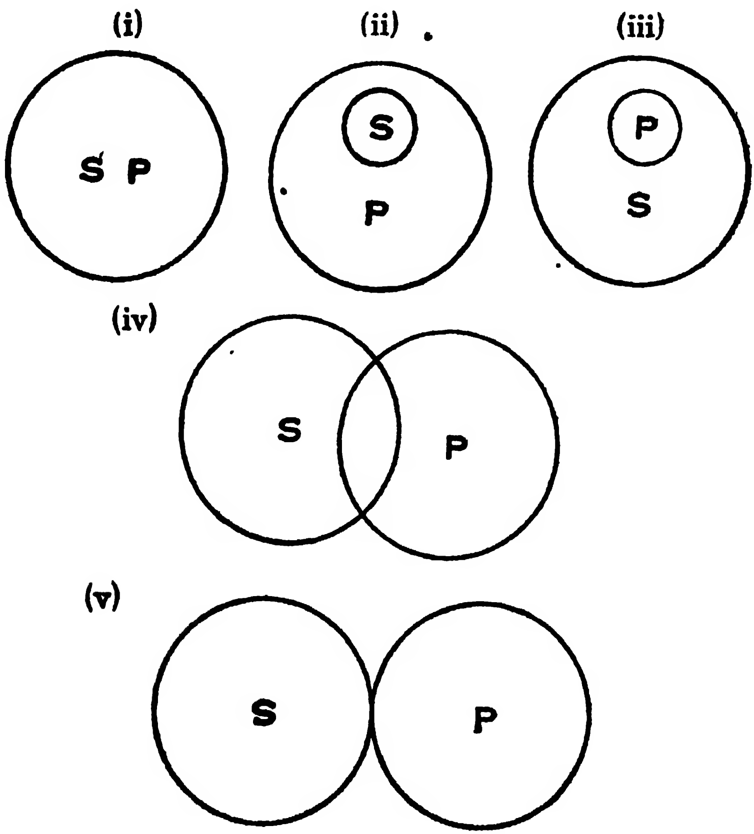
Obversion rightly regarded as a mode of inference.

Obversion of affirmative propositions rests upon the principle of contradiction. If a thing is P, it cannot be not-P. Obversion of negative propositions rests upon the principle of excluded middle, according to which if a thing is not P, it is not-P.

Relation of obversion to principles of contradiction and excluded middle.

Obversion is a reciprocal process, since we can always pass from a proposition to its obverse and back from the obverse to the original proposition. Thus by obverting All S is P we get No S is not-P as the obverse, and if we obvert this again we can return to the original proposition, the obverse of No S is not-P being All S is P. So in obversion the obvertend and the obverse are equivalent propositions. The obverse always means the same thing as the obvertend, though they differ in expression.

**Obvertend and obverse are equivalent propositions.**



Bain speaks of material obversions, *e.g.*, 'Warmth is agreeable' (obvertend), 'cold is disagreeable' (obverse); 'happiness is good' (obvertend), 'misery is bad' (obverse). But such material obversions cannot be regarded as examples of obversion proper, since in them the obvertend and the obverse are both affirmative propositions.

Material obversion is not obversion proper.

The rules of obversion, like those of conversion, may be illustrated by Euler's diagrams (see page 140).

In obversion we want to know the relation between S and not-P.

A propositions are represented by diagrams (i) and (ii), in both of which it is plain that there is nothing in the class S which falls outside the class P; *i.e.*, No S is not-P.

I propositions are represented by diagrams (i), (ii), (iii) and (iv). In (iii) and (iv) *some* S is identical with some P and is therefore not not-P; in (i) and (ii), as we have already seen, No S is not-P, *i.e.*, all (and therefore some) S is other than not-P, *i.e.*, all (and therefore some) S is not not-P. Here we are only justified in stating what is common to all four cases, *viz.*, that some S is not not-P.

E propositions are represented by diagram (v). Here evidently, all that belongs to the class S falls outside the class P; *i.e.*, all S is not-P.

O propositions are represented by diagrams (iii), (iv) and (v). We now see that in (iii) and (iv) part of the class S is not included in the class P, *i.e.*, some S is not-P; in (v) we have already seen that all S is not-P, from which it follows that *some* S is not-P; taking what is common to all these cases, we therefore have that some S is not-P.

**Obverted conversion**

If we *first convert* a proposition and *then obvert* it, we get what is known as *obverted conversion*, which is thus a process of

reasoning involving both conversion and obversion. Let us take the proposition All S is P. Its converse is Some P is S, and its obverted converse Some P is not not-S. Thus the obverted converse of S a P is P o S′. No S is P (original proposition), No P is S (converse), All P is not-S (obverted converse): thus the obverted converse of S e P is P a S′. Some S is P (original proposition), Some P is S (converse), Some P is not not-S (obverted converse): thus the obverted converse of S i P is P o S′. Some S is not P cannot be converted, being an O proposition; having no converse it can have no obverted converse.

## Contraposition

*"Contraposition* may be defined as a process of immediate inference in which from a given proposition another proposition is inferred having for its *subject* the *contradictory* of the *original predicate"* (Keynes). Contraposition is not an independent form of immediate inference, but a combination of obversion and conversion. It rests upon the same fundamental principles of thought upon which obversion and conversion rest. The *conclusion* which is obtained by contraposition is called the *contrapositive*; there is no corresponding name in general use for the original proposition, which however might be (and by some writers is) designated the contraponend. If we first obvert a proposition and then convert it, we get the contrapositive. This process of obversion and conversion is called *conversion by negation* as well as contraposition. Thus the converted obverse of a proposition is called contrapositive, but if we *obvert* this *again* we get what is called the *obverted contrapositive,* or as Keynes calls it, the full contrapositive. What we have designated the contrapositive he distinguishes as the *partial* contrapositive. Older logicians gave the name of contrapositive to what we have called obverted contrapositive or full contrapositive on the ground that since contraposition is a form of conversion, the

Contraposition is a combination of obversion and conversion. Distinction between contrapositive (or partial contrapositive) and obverted (or full) contrapositive.

quality of the premise and of the conclusion should be the same, and that only when the converted obverse is again obverted is the quality of the conclusion the same as that of the original proposition. But we need not restrict the name contrapositive to what we have called obverted or full contrapositive. We may however make a distinction between simple contrapositive and obverted or full contrapositive. Since the quality of the original proposition is the same as that of the full contrapositive, the latter, as distinguished from the simple contrapositive, provides us with symmetry. We shall regard 'simple' or 'partial contrapositive' as synonymous with 'contrapositive'. Now let us see what results contraposition yields.

Contraposition of A.

S a P—All S is P (original proposition or premise), S e P′—No S is not-P (obverse), P′ e S—No not-P is S (contrapositive, or converted obverse or partial contrapositive), P′ a S′—All not-P is not-S (obverted contrapositive or full contrapositive). Let us illustrate the above by a concrete example: All whales are warm-blooded (original proposition or premise), No whales are not-warm-blooded (obverse), No not-warm-blooded creatures are whales (contrapositive), All not-warm-blooded creatures are other than whales (obverted contrapositive or full contrapositive).

Contraposition of E.

S e P—No S is P (premise), S a P′—All S is not-P (obverse), P′ i S—Some not-P is S (contrapositive), P′ o S′—Some not-P is not not-S (obverted contrapositive or full contrapositive). As a concrete example take—No horses are carnivorous (original proposition), All horses are non-carnivorous (obverse), Some non-carnivorous creatures are horses (contrapositive), Some non-carnivorous creatures are not other than horses (full contrapositive).

Contraposition of O.

S o P—Some S is not P (premise), S i P′—Some S is not-P (obverse), P′ i S—Some not-P is S (contrapositive), P′ o S′—Some not-P is not not-S (full contrapositive). This may be illustrated by the following example:—Some flowers are not green (pre-

mise), Some flowers are non-green (obverse), Some non-green things are flowers (contrapositive), Some non-green things are not other than flowers (full contrapositive).

I cannot be contraposed.

S i P—Some S is P (original proposition), S o P′—Some S is not not-P (obverse). This, being an O proposition, cannot be converted, and therefore I cannot be contraposed.

Contraposition and obversion are artificial but not on that account useless

Contraposition, like obversion, is artificial, since it has recourse to negative terms. Though this is true, contraposition does represent our actual thinking and therefore is not useless. According to De Morgan, if we know that S is P, we also know that not-S is not-P, and not-P not-S. Therefore he argues that contraposition is useless. We do not deny that every form of immediate inference is simple, and if we have the premise we can easily find out what the conclusion is to be. Yet contraposition is not useless, and we shall discover its utility when we consider inductive inference, in which negative instances are as valuable as positive ones. We should also note that both contraposition and obversion are sometimes able to avoid the use of negative terms. Thus if the premise is 'Anything that is S is P', it may be contraposed as 'Anything that is not P is not S'.

A and O can be contraposed simply, E only by limitation.

Since the contrapositive of S a P is either P′ e S or P′ a S′, and that of S o P is either P′ i S or P′ o S′, A and O are contraposed *simply*, because in them the premise and the conclusion are the same in quantity. But since the contrapositive of S e P is either P′ i S′ or P′ o S′, the premise and the conclusion in the contraposition of E are different in quantity, and therefore the contraposition of E is per accidens or *by limitation*, the premise being universal and the conclusion particular.

### 4. Inversion

Keynes was one of the first to recognise ***Inversion as a distinct form of eduction.*** Previously logicians regarded it as a process of immediate inference not distinct from contraposition. But inversion may rightly be regarded as distinct from contraposition, since the results obtained by the two processes are not the same. We may follow Keynes in defining inversion as "a process of immediate inference in which from a given proposition another proposition is inferred having for its *subject* the *contradictory of the original subject.*" Thus in inversion of a proposition with S as its subject, not-S is to be the subject of the conclusion. To obtain this result we may begin by converting the original proposition, so that we may get S in the predicate; obversion of this new proposition, as we already know, gives us a proposition whose predicate is not-S, and if we can convert this again, we obtain a proposition of which not-S is the subject, which is what we require. We might alternatively start by obverting the original proposition, from which by alternate conversion and obversion we obtain the obverted contrapositive with not-S in the predicate: where conversion of this is possible, we again obtain a proposition whose subject is not-S. We thus find that inversion, like contraposition, is a combination of the two main processes of eduction, obversion and conversion. The original proposition is called the ***invertend*** and the conclusion the *inverse.* Inversion requires that not-S should be the subject of the conclusion, but its predicate may be either P or not-P. When the subject is not-S and the predicate is not-P we have what Keynes calls *full* inversion, but when the subject is not-S and the predicate is P we have what he calls ***partial*** inversion. As the *full* inverse is the obverse of the partial inverse, Welton gives it the name of *obverted* inverse. Welton provides the following rule of inversion, which accords with the description given above—"*Convert* either the *obverted converse* or the *obverted*

General nature of inversion explained. Distinction between partial inverse and full inverse.

*contrapositive.*" Whether we begin with obversion or with conversion depends upon the circumstances of the case, as will be apparent from the examples below.

We proceed to enquire whether A, E, I and O may all be inverted, and if so, by which process.

Inversion of A explained.

Let us see if A can be inverted by beginning with conversion. From S a P—All S is P (premise), we have P i S—Some P is S (converse), and P o S′—Some P is not not-S (obverted converse). This being an O proposition cannot be converted again. Thus A cannot be inverted if we begin with conversion. Let us see what happens if we begin with obversion. We have S a P —All S is P (premise); S e P′—No S is not-P (obverse); P′ e S— No not-P is S (contrapositive); P′ a S′—All not-P is not-S (obverted contrapositive); (by conversion) S′ i P′—Some not-S is not-P (obverted inverse or full inverse); (by further obversion) S′ o P—Some not-S is not P (inverse or partial inverse). The following is a concrete example: All virtuous men are happy (premise): No virtuous men are non-happy (obverse); No non-happy men are virtuous (contrapositive); All non-happy men are non-virtuous (obverted contrapositive); Some non-virtuous men are non-happy (obverted inverse or full inverse); Some non-virtuous men are not happy (inverse or partial inverse).

Inversion of E explained

Let us now see what happens if we try to invert E by beginning with obversion. From S e P—No S is P (premise), we have S a P′—All S is not-P (obverse); P′ i S—Some not-P is S (contrapositive); P′ o S′—Some not-P is not not-S (obverted contrapositive). This being an O proposition cannot further be converted. So we cannot invert E if we begin with obversion. Let us therefore begin with conversion. Then we have: S e P—No S is P (premise); P e S—No P is S (converse): P a S′—All P is not-S (obverted converse); S′ i P—Some not-S

is P (inverse or partial inverse); S′ o P′—Some not-S is not not-P (obverted inverse or full inverse). The following is a concrete example: No cruel man is praiseworthy (premise); No praiseworthy man is cruel (converse); All praiseworthy men are non-cruel (obverted converse); Some non-cruel men are praiseworthy (inverse or partial inverse); Some non-cruel men are not other than those who are praiseworthy (obverted inverse or full inverse).

I and O proposi tions cannot be in verted.

I and O *cannot be inverted.* S i P if obverted becomes S o P′, but S o P′ cannot be converted, being an O proposition. Again S i P if converted becomes P i S, which when obverted becomes P o S′; but this again, being an O proposition, cannot be converted. So I cannot be inverted by either method. S o P, being an O proposition, cannot be converted. If we begin with obversion, it becomes S i P′, which being converted becomes P′ i S; P′ i S obverts to P′ o S′, but this, being an O proposition, cannot be converted again. Thus O cannot be inverted at all.

In conclusion we may point out that the partial inverse of A being S′ o P (some not-S is not-P), P is distributed in the conclusion, as it is the predicate of a negative proposition, though it is not distributed in the premises S a P (All S is P). Thus the inversion of A apparently involves a flaw. But the inverse S′ o P is obtained by obversion and conversion, which are valid processes of reasoning, and therefore the inversion of A is regarded by Keynes as legitimate. But this view is difficult to accept. It should also be observed that both A and E invert to I and O. Accordingly, *inversion* is *always* per accidens or *by limitation.*

## Eductions of Hypothetical Propositions

Hypothetical propositions, when they are to be converted, obverted, contraposed or inverted, should be expressed in their denotative form. Thus hypothetical propositions, when they are

reduced to the denotative form and are truly conditional, provide us with the four traditional forms, *viz.*, If any S is M, then always that S is P (A); If any S is M, then never that S is P (E); If an S is M, then sometimes that S is P (I); If an S is M, then sometimes that S is not P(O).

Educations of hypothetical propositions.

Let us now see how the processes of eduction apply to hypothetical propositions. We first give a list of eductions from A:—

If any S is M, then always, that S is P (premise).
If any S is M, then never, that S is not P (obverse).
If an S is P, then sometimes, that S is M (converse).
If an S is P, then sometimes not, that S is not M (obverted converse).

If any S is not P, then never, that S is M (contrapositive or partial contrapositive).

If any S is not P, then always, that S is not M (obverted contrapositive or full contrapositive).

If an S is not M, then sometimes not, that S is P (partial inverse or inverse).

If an S is not M, then sometimes, that S is not P (full inverse or obverted inverse).

Eductions from E:—
If any S is M, then never, that S is P (premise).
If any S is M, then always, that S is not P (obverse).
If any S is P, then never, that S is M (converse).
If any S is P, then always, that S is not M (obv. conv.).
If an S is not P, then sometimes, that S is M (contrapositive or partial contrapositive).

If an S is not P, then sometimes not, that S is not M (obv. contrapositive or full contrapositive).

If an S is not M, then sometimes, that S is P (partial inverse or inverse).

If an S is not M, then sometimes not, that S is not P (full inverse or obv. inv.).

Eductions from I:—

If an S is M, then sometimes, that S is P (premise).

If an S is M, then sometimes not, that S is not P (obverse).

If an S is P, then sometimes, that S is M (converse).

If an S is P, then sometimes not, that S is not M (obv. conv.).

Eductions from O:—

If an S is M, then sometimes not, that S is P (premise).

If an S is M, then sometimes, that S is not P (obverse).

If an S is not P, then sometimes, that S is M (contrapositive or partial contrapositive).

If an S is not P, then sometimes not, that S is not M (obv. contrapositive or full contrapositive).

## Other Eductions

There are certain cases of immediate inference which cannot strictly be regarded as formal, and cannot be expressed by symbols. Such inferences have been called *material* immediate inferences as distinct from formal immediate inferences. We may consider these eductions now. Among the forms of immediate inference which we are now going to consider there are three *material forms* and the remaining two, namely, *change of relation* and *modal consequence* may be regarded as more or less formal. We shall first take up the material forms of immediate inference and then consider the two remaining forms of formal inference.

### 1. Inference by Added Determinants

In inference by added determinants we *add* the *same qualification* to both the *subject* and the *predicate* of a proposition, and hold the result of our operation to be true, on the strength of the truth of the original proposition. According to Keynes, "Immediate inference by added determinants is a process of immediate inference which consists in limiting both the

subject and the predicate of the original proposition by means of the same determinant." All P is Q, ∴ All AP is AQ. A negro is a fellow creature; therefore a suffering negro is a suffering fellow-creature. A mango is a fruit; therefore a sweet mango is a sweet fruit. Leibnitz expresses this inference by mathematical symbols: *e.g.*, "If A=B and L=M, then A+L=B+M". This is admissible, provided that the sign (=) is not taken to signify equality but merely to denote the logical copula 'is', and that the sign (+) simply implies a connexion of elements, and not a mathematical addition of units. Keynes gives a proof of the formal validity of the inference, P is Q, ∴ AP is AQ. Thus: AP is a sub-class of P, but P is Q, ∴ AP is Q. But AP is also A; *i.e.*, AP is both A and Q; *i.e.*, AP is AQ. But the truth of the inference by added determinants, as it depends upon knowledge of facts, cannot properly be expressed by symbols, the truth of which holds good universally. We cannot pass from the proposition 'An ant is an animal' to the conclusion that 'A large ant is a large animal'. Similarly we cannot conclude that 'a good teacher is a good man' from the premise that 'a teacher is a man'. From the above discussion it is clear that the validity of inference by added determinants rests upon a knowledge of facts and as such it is a material form of immediate inference.

## 2. Immediate Inference by Complex Conception

In this kind of inference. "the *subject* and *predicate* of a given proposition are used to *qualify* in some way the *same term.* and thus *complex concepts* are formed that are made *subject* and *predicate* of a *new proposition*". Keynes says, "Immediate inference by complex conception is a process of immediate inference which consists in employing the subject and the predicate of the original proposition as parts of a more complex conception", *e.g.*, Physics is a science, therefore physical treatises are scientific treatises. The dog is an animal. therefore the head of a dog is the head of an animal. In such inferences the subject

and the predicate of the original proposition become adjectival in the conclusion, and form parts of more complex conceptions. Thus according to this mode of inference, if P is Q, it follows that whatever stands in a certain relation to P stands in the same relation to Q.

The validity of immediate inference by complex conception, like that of immediate inference by added determinants, does not rest upon purely formal grounds but upon knowledge of concrete facts. Thus from the proposition 'A cat is not a dog' we cannot pass on to the conclusion that 'the owner of a cat is not the owner of a dog'. Similarly from the proposition 'Horses are animals' we cannot pass on to the conclusion that 'The greater number of horses is the greater number of animals'. Similarly we cannot say that since Protestants are Christians, the majority of Protestants are the majority of Christians.

Material eductions ought to be treated of in the special branch of logic known as the logic of relatives, and not in connection with formal immediate inferences. Material inference shows that "arguments are not all built on the principle of American watches, with interchangeable parts, so that terms from one may be transferred to another without interfering with the working of the inference; and that the study of the inference, like the study of life, is largely a matter of examining types, though there are a certain number of common forms, which recur identically in diverse contents". Even in studying syllogism, which is the ideal type of formal inference, we cannot ignore the consideration of facts.

### 3. Immediate Inference by Converse Relation

This process is *analogous* to ordinary *conversion*, but properly belongs to the logic of relatives. In such an inference we pass from one statement of relation to another, that is, from the relation in which S stands to P to the relation in which P stands to S. Thus the subject and the predicate of the original

proposition are transposed in the conclusion. Examples are: S is greater than P, therefore P is less than S. S is to the right of P, therefore P is to the left of S. S is to the east of P, therefore P is to the west of S. Alexander is the son of Philip, therefore Philip is the father of Alexander. Sita is the wife of Ram, therefore Ram is the husband of Sita; etc. Such inferences are not possible without knowledge of the implications of the relations involved. The distinction between formal and material inference is not absolute, but is made for the sake of convenience. The kinds of material inference illustrated above are treated of in the logic of relatives.

Two other usually recognised forms of immediate inference may be discussed briefly.

(4) *Immediate Inference by Change of Relation* is the process whereby we pass from a categorical to a hypothetical or a disjunctive proposition, or from a hypothetical to a disjunctive or a categorical proposition, or from a disjunctive to a hypothetical or a categorical proposition, *e.g.*, All S is P (categorical), therefore If anything is S it is P (hypothetical). Rainy weather is wet weather, therefore If the weather is rainy, it is wet. Every S is either P or Q (disjunctive), therefore Any S that is not P is Q (categorical). Every man is either intelligent or non-intelligent, therefore Every man being not intelligent is non-intelligent. Every flower is either white or not-white (disjunctive), therefore If any flower is not white it is not-white (hypothetical). Similarly, If a man is honest he is trusted (hypothetical), therefore A man being honest is a man who is trusted (categorical). We need not give more examples. But a word of caution is necessary; in passing from a proposition of one relation to a proposition of another relation, the meaning of the original proposition is liable to suffer distortion, so care must be taken to avoid it.

(5) *Immediate Inference by Modal Consequence or Inference by Change of Modality* is analogous to subaltern inference. In this variety of immediate inference we can pass from the validity

of the apodeictic or necessary judgment to the validity of the assertoric, and from that to the validity of the problematic judgment, but not *vice versa.* On the other hand, from the invalidity of the problematic judgment we can pass to the invalidity of the assertoric and necessary judgment, but not *vice versa.* Let us take the three propositions, S must be P (necessary), S is P (assertoric), S may be P (problematic). If the proposition S must be P is true, then the proposition S is P is true, and this being true, the proposition S may be P is also true. But if the proposition S may be P is true, we cannot conclude that S is P, much less that S must be P. Again if the proposition S may be P is false, then the propositions S is P and S must be P are also false. But if the proposition S must be P is false, then the other two propositions, S is P and S may be P, need not be false.

# CHAPTER XI

## PURE SYLLOGISMS

### Definition and Nature of Syllogism

Syllogism regarded as the most important form of deductive or formal inference.

*Syllogism* is the most important form of deductive inference. We have already discussed the different forms of immediate inference, and it is undeniable that the elements of inference involved therein are not very important. Some therefore have refused to regard immediate inference as inference proper. But we have pointed out that immediate inference involves some, even if only slight, movement of thought from the known to the unknown, or, at any rate, from that which is explicitly known to that which was only implicitly known, and therefore it should be regarded as inference. Syllogism on the other hand has, from the time of Aristotle, been regarded as the ideal form of deductive or formal inference. No one has ever refused to call it inference, though some logicians, such as Mill and his followers, have questioned the capacity of syllogistic inference to establish valid conclusions without the aid of induction. In a *syllogism* we draw a *conclusion* from *two premises,* not from one premise only as in immediate inference. Every syllogistic reasoning requires *three* and only three *propositions,* two of which, called *premises* or *implicants,* are *given;* and the other is called the *conclusion* or the *implicate.* Further, the syllogism requires three and only *three terms.* A syllogism may be regarded either as an *implication,* in which case the propositions are called the implicants and the implicate, or it may be regarded as an *inference,* in which case the propositions are called the premises or data and the conclusion. Syllo-

Syllogism a demonstrative form of inference.

gism is one of the *demonstrative* forms of inference, for within its limits its validity cannot be questioned; that is, from the formal point of view, the truth that it establishes on the basis of given truths has demonstrative certainty. The etymological meaning of the term syllogism is 'collecting together', and this meaning is significant inasmuch as the elements of a syllogism are thought of together.

Syllogism defined and its nature explained.

*Syllogism* has been defined by Johnson as "an argument containing *two premises* and a *conclusion, involving* between them *three terms, each* of which occurs in *two* different propositions". According to Joseph, "A syllogism is actually an argument in which, from the given relation of two terms, *in the way of subject and predicate,* to the same third term, there follows necessarily a relation, *in the way of subject and predicate,* between those two terms themselves". If we take the syllogism All M is P, All S is M, therefore All S is P, we find that it consists of three propositions, of which the first two are the premises and the last one is the conclusion; and contains three terms, P, M and S. The term which is the *predicate* of the *conclusion* is called the *major term,* that which is the *subject* of the *conclusion* is called the *minor term,* and that which occurs in the *premises* but *not* in the *conclusion* is called the *middle term.* In the above example P is the major term, S the minor term and M the middle term. The proposition in which the *major term* occurs is called the *major premise,* and that in which the *minor term* occurs is called the *minor premise;* in the conclusion both the major and the minor terms are present. The *major* and the *minor* terms are called the *extremes,* because they form the extremities of the conclusion. The relation between M and P, M and S, and S and P is in the way of subject and predicate. In the premises, S and P are related to the third term M in the way of subject and predicate, and in the conclusion S and P are related in the same way.

The *middle* term *mediates* the conclusion by *connecting* the *major* and the *minor* terms. According to Bosanquet, the middle term may be regarded as the "copula or grip which holds the conclusion together, made explicit and definitely stated". It was supposed by older logicians that the major term has the largest extent, the minor term the least extent, and the middle term an extent intermediate between the two. In a syllogism in which all the propositions are universal affirmatives, *i.e.*, A propositions (as in the above example), the extent of the major term is larger than that of the middle term, and that of the middle term is larger than that of the minor. But in a negative syllogism this does not hold good. Take the syllogism, 'No men are monkeys, John is a man, therefore John is not a monkey'. Here we cannot say whether the extent of the major term is larger or smaller than, or equal to, that of the middle term. The *middle* term is so called because it *establishes a connection* between the major and the minor terms, and thus mediates the conclusion.

Function of the middle term.

The examples show that each of the three terms of a syllogism occurs twice in an inference. The major term occurs in the major premise and the conclusion; the minor term occurs in the minor premise and the conclusion; the middle term occurs in both the premises but not in the conclusion.

Usually the major premise is stated first and then the minor premise, and the conclusion at the end. But in actual reasoning this order may not be strictly followed, nor is the order of the syllogism important. We cannot agree with Jevons when he makes the observation that the cogency of a syllogistic argument becomes apparent if the minor premise is stated first. The usual order is satisfactory, inasmuch as the major premise is generally universal and provides the general case of which the minor premise is but a particular application. Though the order

Order of propositions in a syllogism not important.

of the propositions in a syllogism has no logical significance, it may be of some rhetorical importance. The older logicians stated the conclusion, which they called the question, first, and the premises, which they called the reason, afterwards. The order followed was—

Socrates is mortal (Question)

because

$$\left.\begin{array}{r}\text{All men are mortal}\\ \text{and Socrates is a man}\end{array}\right\}\text{(Reason).}$$

It was also customary to call the major premise the principle or proposition, the minor premise the reason or assumption, and the conclusion the deduction or collection.

Syllogism a type of mediate inference, in which the conclusion is never more general than the premises.

Syllogism is mediate deductive inference, in which the *conclusion* can *never* be *more general* than the *premises.* The truth of the conclusion follows from the truth of the premises. According to Welton and Monahan, "The whole force of a syllogism depends upon the necessity with which the inferred proposition follows from those given as data, and this necessity must be evident from the mere form of the argument". The conclusion of a syllogism, being derived from the premises, is not absolutely a new proposition. Though it follows necessarily from the premises, nevertheless it goes beyond them; but in formal inference the element of necessity is more important than the element of novelty. Syllogistic inference is similar to the multiplication of large numbers, in which the product, though a new number, is implied in the data and follows from them as a matter of course. In formal inference, we do not question the validity

In formal inference the element of necessity is more important than the element of novelty.

of the premises, which are accepted as true, and our problem is to derive a conclusion from them in conformity with the rules of the syllogism. If the premises are materially true, the conclusion also is materially true; *e.g.*, 'All birds have wings, This sparrow is a bird, This sparrow has wings'. It may so happen that the premises are materially false while the conclusion is materially true. But this is accidental, and is a case of mere coincidence, not of necessity. For example, in, 'All birds are rational, Men are birds, ∴ Men are rational', the conclusion is materially true, though the premises are materially false.

The *terms* of a syllogism are called its *remote matter,* the *propositions* its *proximate matter.* "The matter of a syllogism is given in its terms, which vary according to the subject to which the argument refers. Its form consists in that relation of the terms by which they are united in two propositions necessitating a certain conclusion" (Welton).

The opposite of syllogism is antilogism, the propositions of which cannot be true together.

. The opposite of the syllogism is the *Antilogism.* Whereas in a syllogism all the three propositions must be consistent with one another and true together, in an antilogism the *propositions cannot be true at once.* Johnson gives the following example of the antilogism: 'All tactful persons sometimes lie; Lord Grey is a tactful person; Lord Grey never lies'. If the propositions 'Lord Grey is a tactful person' and 'Lord Grey never lies' are true, then the proposition 'All tactful persons sometimes lie' cannot be true. Again if the propositions 'All tactful persons sometimes lie' and 'Lord Grey is a tactful person' are true, then 'Lord Grey never lies' cannot be true. In the same way if the propositions 'All tactful persons sometimes lie' and 'Lord Grey never lies' are true, then 'Lord Grey is a tactful person' cannot be true. Thus in an antilogism the propositions are together incompatible.

We may in passing remark that the *syllogism*, being a type of *formal* inference, can be represented by *symbols*.

Pure and mixed syllogisms contrasted.

Syllogisms may be either *pure* or *mixed*. *Pure* syllogisms are those in which the *propositions* are of the *same relation*. Thus in a pure categorical syllogism all the propositions are categorical, in a pure hypothetical syllogism all the propositions are hypothetical, and in a pure disjunctive syllogism all the propositions are disjunctive. All pure syllogisms, whether categorical, disjunctive, or hypothetical, are subject to the same rules, and the distinction between them is not important. *Mixed* syllogisms are (1) *Mixed hypothetical*, also called hypothetico-categorical, (2) *Mixed disjunctive*, also called disjunctive-categorical, and (3) the *Dilemma*. The different forms of the mixed syllogism being subject to different rules, the distinction between them is important. In a *mixed hypothetical* syllogism, the *major* premise is *hypothetical* and the *minor* premise *categorical*. In a *mixed disjunctive* syllogism, the *major* premise is *disjunctive* while the *minor* premise is *categorical*. In a *dilemma*, the *major* premise is a compound *hypothetical* and the *minor* premise is *disjunctive*. In this and the following chapter, we shall discuss the different forms of pure syllogisms, and shall devote a separate chapter to the consideration of the forms of mixed syllogisms.

Pure Syllogisms
1. Pure categorical;
2. Pure hypothetical

Mixed Syllogisms
1. Mixed hypothetical or hypothetico-categorical;
2. Mixed disjunctive or disjunctive-categorical;
3. Dilemma.

## Axioms of Pure Syllogisms

We are now in a position to state and explain the axioms and general rules of syllogisms. We must not suppose that these axioms and general rules are presupposed in syllogistic inference; rather they are deductions from the nature of syllogistic reasoning. These principles are helpful, inasmuch as they enable us to test the correctness of different pieces of syllogistic reasoning. In syllogistic reasoning, we must remember, we are concerned not with the validity of premises and conclusion as such but with the validity of the whole process of reasoning involved. Therefore it has been called the logic of consistency, as opposed to inductive logic, which is supposed to be the logic of truth. But we shall find in the course of our discussion that a good deal of induction rests upon purely formal grounds, though at the same time it depends upon observation of, and experiment upon, facts.

**Axioms and rules of syllogism are deductions from the nature of syllogistic reasoning.**

All reasoning, including syllogistic inference, depends upon the fundamental *laws of thought*. Every *affirmative categorical* syllogism rests upon the principle of *identity*. Take the syllogism, 'All organisms are mortal, Human beings are organisms, therefore Human beings are mortal'. In this argument the middle term 'organisms' is identical with 'mortal' in one premise and with 'human beings' in the other, and hence we can establish a relation of identity between 'human beings' and 'mortal'. By 'identity' however we here mean not absolute identity, but identity amidst diversity. *Negative categorical* syllogisms rest upon the principle of *contradiction*. In the argument, 'No negroes are white, Joseph is a negro, therefore Joseph is not white', the major premise asserts a separation between the middle term 'negroes' and the major term 'white,'

**Affirmative categorical syllogisms rest on the principle of identity, negative ones on the principle of contradiction; pure hypothetical syllogisms require also the principle of sufficient reason.**

while the minor premise asserts a relation of identity between the middle term and the minor term 'Joseph,' and therefore the conclusion asserts a relation of exclusion between the major term and the minor term. Thus in an affirmative syllogism the identity between the extremes is established through the medium of the middle term, while in a negative syllogism the relation of exclusion is established between the extremes, because the middle term in one of the premises excludes one of the extremes, while in the other premise it is connected with the other extreme by a relation of identity. A *pure hypothetical* syllogism depends as much upon the principle of *sufficient reason* as upon the principles of identity and contradiction. 'If any man is a philanthropist, he is loved by many; If any man serves other men in a disinterested manner, he is a philanthropist; therefore If any man serves other men in a disinterested manner, he is loved by many,' is an example of a pure hypothetical syllogism which rests upon the principle of identity and that of sufficient reason.

Axioms of categorical syllogisms given by Whately and Hamilton.

Various *axioms* of syllogisms have been proposed by logicians. The most famous is the 'Dictum de omni et nullo' of Aristotle, which was supposed by him to be the ground of all syllogistic reasoning. But the *axioms* of syllogisms proposed by philosophers are *not fundamental*, because they may be *deduced* from the fundamental *laws of thought*, such as the law of identity, the law of contradiction, etc. They are therefore to be regarded as *middle axioms* derived from the fundamental ones. Before explaining Aristotle's dictum, we may state the axioms proposed by Whately and Hamilton. Whately gives the following two axioms upon which, according to him, all syllogistic reasoning is based: (a) "If two terms agree with one and the same third, they agree with each other." (b) "If one term agrees and another disagrees with one and the same third, these two disagree with each other." Axiom (a) is supposed to be the basis of affirmative syllogisms, axiom (b) that of negative syllogisms. Hamilton gives the following axiom, which he calls "the supreme canon of categorical syllogisms": "In so far as two notions (notions proper and individuals) either both agree, or one agreeing the other does not agree, with a common third notion, in so far these notions do or do not agree with each other." These axioms of Whately

and Hamilton are unassailable, but they cannot be regarded as supreme, as they are derived from the principles of identity and contradiction, of which they are only developed statements. They should not be called canons (*i.e.*, rules), but middle axioms. There is no essential difference between the axioms of Whately and that of Hamilton. The only difference is that Whately uses the terminology of the nominalists, while Hamilton uses that of the conceptualists. Hamilton's single axiom sums up the two axioms of Whately.

Aristotle's dictum explained.

The axiom given by Aristotle is known as the *Dictum de omni et nullo,*[1] which is rendered in English by Welton and Monahan as—"Whatever is distributively predicated, whether affirmatively or negatively, of any class, may be predicated in like manner of anything which can be asserted to belong to that class." The empty scheme of this axiom is M—P, S—M, S—P. This axiom requires that the middle term should be the subject of the major premise and the predicate in the minor. It also requires that the major premise should be universal and the minor affirmative, and that the conclusion should be affirmative if the major premise is affirmative, and negative if the major premise is negative. The dictum also requires that the middle term must be distributed in the major premise. Aristotle and his followers supposed that if a syllogistic reasoning does not appear in the form required by the dictum, it should be reduced to that form in order that its validity may be proved. We may point out that Aristotle's axiom, like those of Hamilton and Whately, is the development of the two fundamental laws of thought, *viz.*, the law of identity and the law of contradiction, and should like them be regarded as a middle axiom. We shall find in the course of our discussion that a syllogistic reasoning may be valid even though it does not appear in the form required by the dictum.

Mill finds fault with the dictum of Aristotle, which rests upon the class-inclusion theory of predication; that is, according to Aristotle the terms of every proposition are to be read in denotation. Mill argues that

[1] De omni et nullo, literally = 'Concerning all and none'.

general terms are usually read in connotation, and denotation depends upon connotation. Therefore the dictum of Aristotle has to be modified. So according to Mill 'Nota notae' (lit., 'mark of a mark') is a better dictum. This may be rendered in English either as (1) "Whatever has any mark, has that which it is a mark of", or as (2) "Whatever is a mark of any mark, is a mark of that which this last is a mark of." The first reading applies well to such a syllogism as, 'All men are mortal, Socrates is a man, therefore Socrates is mortal.' Here the major premise implies that the attributes of men are associated with those of mortals, the minor premise implies that Socrates possesses human attributes, and the conclusion implies that Socrates possesses the attributes of mortality. In other words Socrates has the mark of a man, and man has the mark of mortality or of being mortal, and therefore Socrates has the mark of mortality. The second reading of the dictum is suited to syllogisms in which both the premises are universal, *e.g.*, 'All men are mortal, kings are men, therefore kings are mortal.' In this case the major premise implies that the attributes of men are associated with the attribute of mortality, the minor premise implies that the attributes of kings are associated with those of men, and the conclusion implies that the kingly attributes are associated with the attribute of mortality. In other words, the attributes of kings are a mark of those of man, the attributes of man are a mark of the attribute of mortality, and therefore the attributes of kings are a mark of the attribute of mortality.

Explanation of the dictum Nota Notae, which is preferred by Mill.

## General Rules or Canons of Syllogisms

Logicians generally recognise *six general rules of syllogism* and three *corollaries* from them. Of the six rules, two refer to the *nature* of the syllogism, two to the *distribution of terms* in it, and the remaining two to the *quality* of the *propositions* of the syllogism. We may deduce them all from the dictum of Aristotle explained above, by a slight generalisation of it. We may now explain these rules:—

I. A syllogism must contain *three* and only three *terms.*

II. A syllogism must consist of *three* and only three *propositions.*

These two rules serve to *define* the syllogism as a particular

form of argument. They are *not* required to prove the *validity* of a syllogistic argument. We know that in syllogistic reasoning we establish a relation between two terms in the way of subject and predicate, because these terms are related to a common third term in the premises in the way of subject and predicate. So in a syllogism there can be only three terms. Similarly a syllogism requires that a conclusion should be drawn from two and only two premises, and therefore there cannot be more or less than three propositions in a syllogism. An argument may be valid if it contains more or less than three terms, or more or less than three propositions, but such an argument is not a syllogism. Thus we have seen that an immediate inference consists of only two propositions, yet it is a valid form of reasoning. In the same way a train of syllogisms may be a valid form of reasoning, though it contains more than three terms and more than three propositions.

The first two rules serve to define the syllogism.

If one of the terms of a syllogism is ambiguous, then it becomes equivalent to two terms, and strictly speaking the syllogism then presents us with four terms instead of three. If any term of a syllogism is ambiguous, it commits the *fallacy of four terms,* technically known as Quarternio Terminorum. Ambiguity of the middle term is frequent in syllogistic reasoning, and in this case the fallacy is commonly known as the *fallacy of ambiguous middle.* Similarly there may be the fallacy of ambiguous major or of ambiguous minor. The function of the middle term is to establish a relation between the two extremes, and if it is ambiguous, it fails to discharge its proper function. It is an axiom of logic that in the same argument the same term must be used in the same sense wherever it occurs. The violation of this axiom leads to fallacious reasoning. The following two arguments show how,

If any of the terms of a syllogism is ambiguous, then there is the Fallacy of Four Terms or Quarternio Terminorum.

through the ambiguity of the middle term of a syllogism, fallacy is committed. (1) De Morgan gives the following example: All criminal actions ought to be punished by law; prosecutions for theft are criminal actions; therefore Prosecutions for theft ought to be punished by law. In this case the middle term 'criminal action' is ambiguous: in the major premise it means the committing of a crime, while in the minor it means a form of legal procedure. (2) The second example to illustrate the fallacy of ambiguous middle is provided by Joseph: No vegetable has a heart; a good lettuce has a heart; therefore a good lettuce is not a vegetable. In this case the conclusion is fallacious, because the middle term 'heart' in the major premise means a particular kind of organ for maintaining the circulation of the blood, and the same term in the minor premise means the central part of the lettuce. In the chapter on deductive fallacies we shall illustrate the fallacy of ambiguous major and of ambiguous minor.

Deduction of the first two rules from the dictum of Aristotle.

We have stated that the general rules of syllogism can be deduced from the dictum of Aristotle. The dictum provides for three and only three terms and three and only three propositions. It says that whatever is predicated of a class can be predicated of anything belonging to that class. Thus whatever is predicated furnishes the major term, the class of which it is predicated is the middle term, and that which is included in the class is the minor term. Further, what is predicated of a class is stated by the major premise, that something belongs to the class is stated in the minor premise, and the conclusion predicates of that something what was originally predicated of the class.

The third rule explained.

III. The *middle term must be distributed at least in one of the premises.*—The function of the middle term is to connect the extremes, and therefore it must have identical reference in the two pre-

mises; but this can be secured only if it is distributed at least once. If the middle term is not distributed in either of the premises, then the part of its extension to which the major term is related may be different from the part of its extension to which the minor term is related. If this happens, no connection can be established between the extremes. When the middle term is *not distributed* in *either* of the premises, we have the *fallacy of undistributed middle.* Thus we cannot draw any conclusion from the premises 'Some animals are herbivorous' and 'Some animals are carnivorous.' Similarly from the premises 'Some Indians are Hindus' and 'Some Indians are Moslems', no conclusion can be deduced. If a conclusion is drawn from such premises, it must be fallacious. This rule can also be proved by means of Euler's diagrams. But Euler's diagrams rest upon the assumption that the terms of a syllogism are always read in extension.

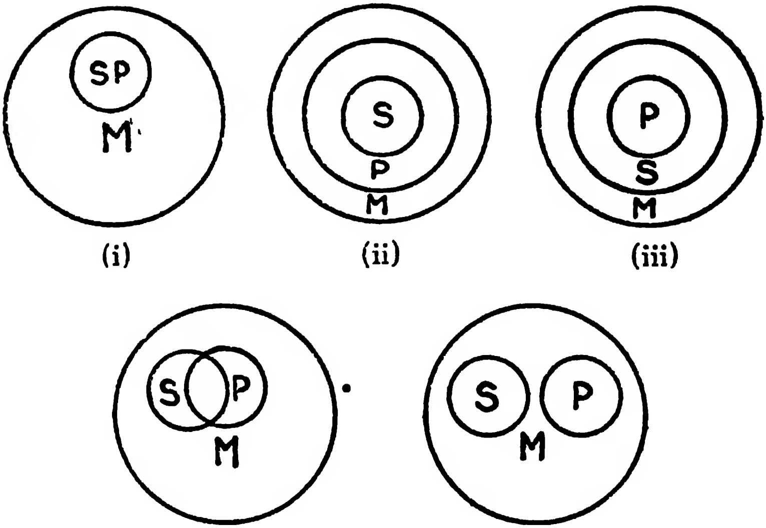
The violation of this rule leads to the fallacy of undistributed middle.

As stated above, the rule that the middle term must be distributed in one of the premises may be illustrated by Euler's diagrams.

For, suppose we are given two premises such as

All P is M,
All S is M,

in neither of which the middle term M is distributed (since it is here in each case the predicate of an affirmative proposition). Then the relations between S, M, and P require the following five diagrams to represent them:




(i) (ii) (iii)

(iv) (v)

If we now look at the circles representing S and P, we find that all the five possible relationships between them are exemplified, and there can therefore be no conclusion, since any proposition stating a relation between S and P excludes at least one of the five relationships expressed by Euler's five diagrams.

Hence if the middle term is not distributed in either premise, there can be no conclusion.

The fourth rule explained.

IV. *No term* may be *distributed* in the *conclusion* which is *not distributed in one of the premises.*—Syllogism requires that the conclusion should not be more general than the premises, and if a term is distributed in the conclusion without being distributed in the premise in which it occurs, there is a likelihood of the conclusion's being more general than is warranted by the premises. If the *major* term is *distributed* in the *conclusion without* being *distributed* in the *major premise,* we have the fallacy of *illicit*

*process of the major,* or *illicit major.* Thus if we draw the conclusion 'No horses are herbivorous' from the premises 'All cows are herbivorous' and 'No horses are cows', this fallacy occurs, because the major term is not distributed in the major premise, where it is the predicate of an affirmative proposition, while it is distributed in the conclusion, where it is the predicate of a negative proposition. If the *minor* term is distributed in the conclusion without being distributed in the minor premise, there is the fallacy of *illicit process of the minor,* or *illicit minor.* Thus if we draw the conclusion 'No mathematicians are capable of virtue' from the premises 'No politicians are capable of virtue' and 'Some mathematicians are politicians,' there is the fallacy of illicit process of the minor term, because it is not distributed in the minor premise, being the subject of a particular proposition, while it is distributed in the conclusion, being the subject of a universal proposition.

Breach of this rule leads to the fallacy either of illicit process of the major or of illicit process of the minor term.

V. *From two negative premises nothing can be inferred.*—If the relation between the middle term and the extremes be denied in the premises, then the conclusion cannot establish a relation between the minor and the major term in the way of subject and predicate. In such a case the middle term cannot establish any connection whatever between the extremes. Only when one of the extremes is connected with the middle term can we, through that connection, infer its agreement with, or separation from, the other extreme. Thus from the premises 'No cows are carnivorous,' 'No horses are carnivorous', no conclusion can be drawn. Similarly from the premises 'No Hindu is a German', 'No Moslem is a German,' no conclusion can be inferred.

The fifth rule explained.

Jevons however argues that this rule cannot be accepted in in its bare form. He gives the following example, in which a

The contention of Jevons that sometimes from two negative premises a conclusion can be inferred does not hold good.

conclusion appears to be drawn from two negative premises: Whatever is not metallic is not capable of powerful magnetic influence; carbon is not metallic; therefore carbon is not capable of powerful magnetic influence. Here the conclusion is valid and it follows from the premises. We may give another similar example. From the premises, 'Whoever is not honest is not happy, Politicians are not honest,' we can draw the conclusion, 'Politicians are not happy.' But if we express these arguments symbolically, we find that they are in the form No not-M is P, No S is M, therefore No S is P. Here we have four terms instead of three. But by obverting the minor premise we obtain a syllogism in the form 'No not-M is P, All S is not-M, therefore No S is P.' This argument is valid, and one of its premises is negative and the other affirmative. Thus we find that the argument given by Jevons does not really start from two negative premises. Again from the premises, 'John is not over 20 years of age, John is not under 20 years of age,' we may draw the conclusion that 'John is 20 years of age.' But this is not an example of syllogistic reasoning. The conclusion in this case follows from a disjunctive major premise, *viz.*, that John is either over 20 years of age or under 20 years of age or 20 years of age, but he is neither over nor under 20 years of age, therefore he is 20 years of age.

This rule also, like rule III, may be illustrated by Euler's diagrams.

Suppose we have given as premises two E propositions—

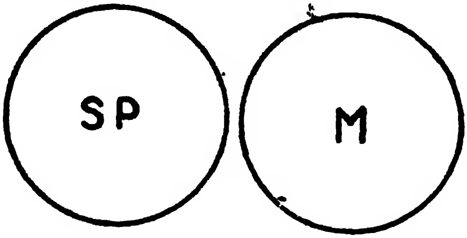
No P is M,

No S is M,

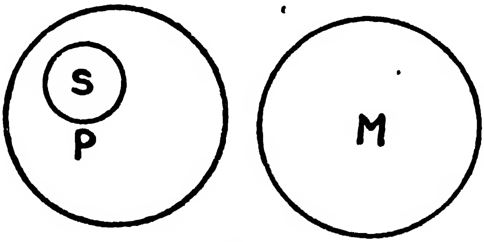
then the relationships between S, M and P are represented by the five diagrams shown hereafter (pp.170-71).

If we consider the circles representing S and P, we find that all the five possible relationships between them are

shown, and there can therefore be no conclusion, since any proposition stating a relation between S and P excludes at least one of the five relationships expressed by Euler's five diagrams.




(i)




(ii)

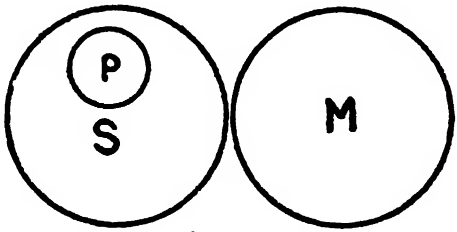
Similarly it may be shown that no other possible pair of negative premises, whether EO or OO, can yield a conclusion as to the relation between S and P, whatever be the order in which the terms are arranged.

VI. If *one premise* is *negative,* the *conclusion* must be *negative,* and to prove a negative conclusion, one of the premises must be negative.—If the middle term is in agreement with one of the extremes and is separated from the other, then the only relation which we can infer between the extremes is one of negation.

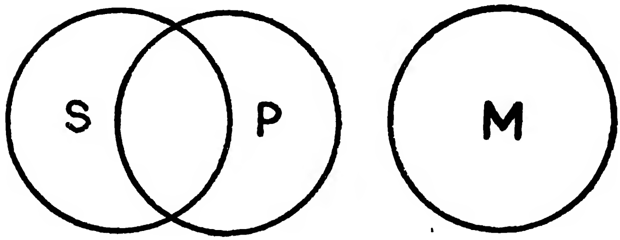
The sixth rule explained.

Thus if M is in disagreement with P while it is in agreement with S, we can only infer that S and P are in disagreement. Thus if the premises be 'A is not equal to B', 'C is equal to B', then the legitimate conclusion is that C is not equal to A. From the premises 'No sheep are carnivorous', 'All my domestic animals

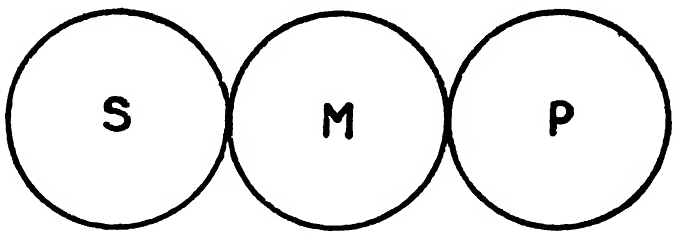
are sheep', we can infer only a negative conclusion *viz.*, 'None of my domestic animals are carnivorous.' In the same way, if the conclusion asserts that the major term and the minor term are in disagreement, that is, a relation of negation exists between them,




(iii)




(iv)




(v)

then such a relation can be inferred only when in the premises one of the extremes is in agreement, and the other in disagreement, with the middle term. If we want to establish the conclusion 'No men are perfect,' we can do so if the premises are 'No animals are perfect' and 'All men are animals'.

This rule can be deduced from the dictum of Aristotle. The dictum says that the major premise may be either affirmative or negative. It also lays down the condition that if the major premise is affirmative, the conclusion must be affirmative, and if it is negative, the conclusion must be negative. Further it requires that the minor premise must be affirmative. So by generalising the dictum we obtain the rule that if one of the premises is negative, the conclusion must be negative, and if the conclusion is negative, one of the premises must be negative.

Deduction of the sixth rule from Aristotle's dictum.

From rules V and VI it naturally follows that when the premises are both affirmative, the conclusion must be affirmative, and to prove an affirmative conclusion both the premises must be affirmative. We have already seen that from two negative propositions nothing can be inferred, that when one of the premises is negative, the conclusion must be negative, and that when the conclusion is negative, one of the premises must be negative. So only when both the premises are affirmative, can the conclusion be affirmative and, when the conclusion is affirmative, both the premises must be affirmative. Aristotle's dictum also provides this in substance. It requires that the minor premise must be affirmative, and that when the major premise is affirmative the conclusion must be affirmative, but when the major premise is negative, the conclusion must be negative. This means that when both the premises are affirmative the conclusion is affirmative, and when one of them is negative the conclusion is negative. We may also point out that when both the premises are affirmative, they assert that the middle term is in agreement with both the extremes. When the premises assert such agreement between the extremes and the middle term, the only conclusion we can infer is that the extremes are in agreement with each other. When A and B both agree with C, we can only infer that they

When both the premises are affirmative, the conclusion must be affirmative, and to prove an affirmative conclusion the premises must be affirmative.

are in agreement with each other. Thus when the premises are 'All violets are white, this flower is a violet,' the legitimate conclusion is that this flower is white. If X and Y are both equal to Z then X and Y are equal to each other. Again when the conclusion asserts that there is agreement between the extremes, the premises from which it is inferred must also assert that each of the extremes is in agreement with the middle term. Thus if the conclusion is A is B, the premises must be affirmative, *e.g.*, C is B and A is C.

The above six rules are more or less independent of one another, but the remaining three rules follow from them and are of the nature of *corollaries.*

Seventh rule explained.

VII. *From two particular premises nothing can be inferred.*—If both the premises are particular, then the possible combinations are I I, OO, IO, OI. If the premises are I I, then they do not distribute any of the terms. But rule III requires the middle term to be distributed at least once in the premises. If the premises are OO, then nothing can be inferred from them, according to rule V, which states that from two negative premises nothing can be inferred. If the premises are either IO or OI, then any conclusion must be negative, according to rule VI, which states that if one of the premises is negative, the conclusion must be negative. If the conclusion is negative, then its predicate P, which is the major term, is distributed in the conclusion. But the fourth rule requires that if a term is distributed in the conclusion, it must be distributed in one of the premises. So if we draw a conclusion from IO or OI, it is necessary to distribute the major term in the major premise. Further, according to rule III, the middle term must be distributed once in the premises. Thus two terms must be distributed in the premises, *viz.*, the major term and the middle term, to warrant a conclusion; but IO or OI distribute between them only one term. If this term is the major term, then there is the fallacy of undistributed middle; if it is the middle term, then the major term remains undistributed in the

premises. Thus if from IO or OI any conclusion is inferred, there is the fallacy either of undistributed middle or of illicit process of the major term. Thus it is proved that from two particular premises nothing can be inferred.

VIII. If *one premise* is *particular*, the *conclusion* must be *particular.*—If one of the premises is particular and the other universal, the possible combinations are AI, AO, IA, OA, EI, EO, IE, OE. Of these EO and OE yield no conclusion because both the premises are negative. AI and IA each distribute only one term, which according to rule III must be the middle term. So neither the major nor the minor term can be distributed in these combinations if the middle term is distributed. Therefore, if the conclusion is universal, the minor term is distributed in the conclusion without being distributed in the minor premise, involving the fallacy of illicit minor. AO, OE, EI, IE each distribute two terms. In each of these combinations, one of the premises being negative the conclusion must be negative; that is, in the conclusion, P, the major term, must be distributed. Therefore in the premises two terms must be distributed, *viz.*, the major term and the middle term. If the conclusion is universal, then the minor term is distributed in the conclusion, and must therefore be distributed in the premises. But each of the above combinations distributes only two terms, and these, as we have pointed out, should be the major and the middle term. Therefore in these combinations there is no room to distribute the minor term in the premises. Therefore the minor term cannot be distributed in the conclusion; that is, the conclusion must be particular. Thus if one of the premises is particular, the conclusion must be particular.

Rule VIII explained.

IX. From a *particular major* and a *negative minor, nothing can be inferred.*—If the minor premise is negative the major must be affirmative, according to rule V. Now if the major premise is particular as well, the major term is not distributed there. But since one of

Rule IX explained.

the premises is negative, the conclusion must be negative, necessitating the distribution of the major term in the conclusion, where it is the predicate of a negative proposition. Thus if a conclusion is drawn from a particular major premise and a negative minor premise, there is the fallacy of illicit major. Thus if the major premise is particular and the minor negative, there cannot be any conclusion.

When the major is particular and the minor negative, the possible combinations are the following: IA, IE, OE, OO. Of these OE and OO yield no conclusion, as both the premises are negative. IO yield no conclusion, both the premises being particular. If a conclusion is drawn from IE it must be negative, in which case the major term is distributed in the conclusion. But in the major premise I, the major term is not distributed. So, if a conclusion be drawn, the fallacy of illicit major is committed. Thus we find that from a particular major and a negative minor nothing can be inferred.

Rules III, IV, V and VI may be regarded as fundamental.

As we have already pointed out, rules VII, VIII and IX are mere corollaries from the other six rules. Rules I and II are purely descriptive, that is, they state what the elements of a syllogism are, and are not rules for proving the validity of a syllogistic reasoning. So the main rules are four in number, *viz.*, two rules for distribution—rules III and IV, which state that the middle term must be distributed once at least in the premises, and that no term should be distributed in the conclusion which was not distributed in the premises; and two rules of quality—rules V and VI, which provide that from two negative premises nothing can be inferred, that if a premise is negative the conclusion must be negative, and that a negative conclusion necessitates a negative premise. We may also remark that rules IV and V may be shown to be involved in rule III, and rule VI in rule V; but we need not enter into intricacies to show how the four main rules imply one another.

Welton states that each of the two following pairs of rules may be regarded as fundamental, *viz.*,—

(1) (a) The middle term must be distributed at least in one of the premises;

(*b*) To prove a negative conclusion requires a negative premise.

(2) (*a*) No term may be distributed in the conclusion which is not distributed in a premise;

(*b*) To prove a negative conclusion requires a negative premise.

Some remarks on the view of Bain and Jevons regarding syllogism.

According to Bain, when a conclusion is inferred from two singular propositions, the argument is not syllogistic. In such a case we can only have a compound proposition. Thus such an argument as 'Plato is the author of the *Republic*, Plato is an Athenian, therefore an Athenian is the author of the *Republic*,' is not really an argument but a compound proposition, *viz.*, 'Plato, who is the author of the *Republic*, is an Athenian', But if this is allowed, then such an argument as 'All men are mortal. All men are rational, therefore Some rational beings are mortal,' may also be regarded as a compound proposition, *viz.*, Men, who are mortal, are rational. Jevons holds that in the conclusion of a syllogism all the terms ought to be stated. Thus from the premises 'Potassium floats on water, Potassium is a metal,' he draws the conclusion, 'Potassium metal is potassium floating on water.' But this view cannot be accepted. The conclusion of a syllogistic reasoning must be a new proposition. Though it follows from the premises, it is not a mere summary of them. Further, in every reasoning there must be some movement of thought from the known to the unknown, but this is not apparent in the interpretation of the syllogism given by Jevons. Further, the middle term should occur only in the premises, and not in the conclusion, in syllogistic reasoning. But Jevons's interpretation does not satisfy this condition. His view that syllogistic reasoning is nothing but substitution cannot. therefore, be conceded.

Figure defined and its forms stated.

*Syllogisms* are distinguished in *figure* according to the *position of the middle term*. "By the figure of a syllogism," says Keynes, "is meant the position of the terms in the premises." Four figures are recognised by logicians. Aristotle however did not recognise the fourth figure, which was first recognised by Galen, the famous physician, and is therefore named

the Galenian figure. In the *first figure* the *middle term* is the *subject* in the *major* premise and *predicate* in the *minor*. The form of this figure is therefore M—P, S—M, S—P. In the *second figure* the *middle term* is the *predicate* in *both the premises,* and its form is P—M, S—M, S—P. In the *third* figure the *middle term* is the *subject* in *both the premises,* and its form is M—P, M—S, S—P. The *fourth figure* is the reverse of the first figure; in it the *middle term* is the *predicate* of the *major* premise and *subject* in the *minor*. The form of this figure is P—M, M—S, S—P.

Mood defined: number of possible moods.

"By the *mood* of a syllogism is meant the *quantity* and *quality* of the *premises* and *conclusion*" (Keynes). According to Joseph, "Syllogisms are distinguished in mood according to the quantity and quality of the propositions composing them." In each figure 16 combinations of premises are possible according to quality and quantity, *viz.*, AA, AE, AI, AO, EA, EE, EI, EO, IA, IE, I I, IO, OA, OE, OI, OO, and each of these yields four possible moods. Thus from AA we may have the moods AAA, AAE, AAI, AAO; similarly from AE we may have four possible moods and so on. Thus in each figure 64 ($16 \times 4$) moods are possible. Therefore in the four figures together the number of possible moods is $64 \times 4 = 256$. Bu we shall find that of these only *24 moods* are *valid,* and others will be found invalid when tested. A mood which is valid in one figure may be invalid in another figure. Thus AAA (All M is P, All S is M, ∴ All S is P) is a valid mood in the first figure. But AAA in the second figure (All P is M, All S is M, ∴ All S is P) is not a valid mood, since it involves the fallacy of undistributed middle. We shall find hereafter that of the 24 valid moods only 19 are fundamental.

## Valid Moods of the First Figure

Every figure has its special rules. The *special rules* of the *first figure* are: (1) the *minor premise* must be *affirmative;* (2) the

The special rules of the first figure stated and proved. They follow from the dictum of Aristotle.

*major premise* must be *universal.* We remember that the form of the first figure is M—P, S—M, S—P. Now if the minor premise is not affirmative, that is, if it is negative, then the major premise must be affirmative (rule V), in which case the major term, as the predicate of an affirmative proposition, will not be distributed in the major premise. But again, if the minor premise is negative, the conclusion must be negative (rule VI), and in it the major term, which is its predicate, must therefore be distributed. Thus if the minor premise is negative, there is the fallacy of illicit process of the major term. Therefore the minor premise must be affirmative. If the minor premise is affirmative, then the middle term, which is the predicate of the minor premise, is not distributed there. It must therefore (by rule III) be distributed in the major premise, of which it is the subject. And to distribute its subject the major premise must be universal.

We may point out that these special rules of the first figure can be deduced from the dictum of Aristotle, which is the basis of the first figure. It provides that whatever is predicated of a class distributively, whether affirmatively or negatively, may be predicated in like manner of anything belonging to that class. Thus according to the dictum, the major premise must make a predication about a class, and therefore it must be universal. The minor premise states that something belongs to that class, and therefore must be affirmative.

Let us now determine the *valid moods* of the *first figure* with the help of the general rules and the special rules of the figure.

The valid moods of the first figure determined.

We have seen that according to quantity and quality sixteen combinations of the premises are possible, *viz.*, AA, AE, AI, AO, EA, EE, EI, EO, IA, IE, II, IO, OA, OE, OI, OO. Of these EE, EO, OE, OO yield no conclusion, according to the general rule V, which states that from two negative premises no

conclusion can be drawn. II, IO, OI yield no conclusion, according to the general rule VII, which states that from two particular premises no conclusion can be drawn. IE yields no conclusion, according to the general rule IX, which states that from a particular major and a negative minor no conclusion can be drawn. Of the remaining combinations, AE, AO, IA, OA are not allowed by the special rules of the first figure. Thus AA, AI, EA, EI alone can yield valid conclusions in the first figure. From AA we may draw two conclusions, *viz.*, A and I. Thus from All M is P, All S is M we may draw the conclusions All S is P and Some S is P. From AI we can draw the conclusion I. Thus from All M is P, Some S is M we obtain the conclusion Some S is P. From EA we may have E and O as the conclusions. The premises No M is P, All S is M may yield the conclusions No S is P and Some S is not P. EI yields the conclusion O. From the premises No M is P, Some S is M we may draw the conclusion Some S is not P. Thus the *valid* moods of the *first* figure are AAA, AAI, AII, EAE, EAO and EIO. Of these, AAI and EAO are called the *weakened* syllogisms of the first figure, because in these cases the premises allow A and E as conclusions respectively. These weakened moods are also called *subaltern* moods, since the conclusions in these cases are subalterns of A and E, which are valid conclusions. So only four moods being fundamental and independent in this figure, some logicians recognise only these four instead of six: *viz.*, AAA, EAE, AII, EIO. These are technically known as Barbara, Celarent, Darii and Ferio respectively. In each of these mnemonic words the vowels represent the propositions of the syllogism. We may now illustrate the four main moods of the first figure by concrete examples:—

1. AAA (Barbara)—All lions are tawny, This animal is a lion, ∴ This animal is tawny.

2. EAE (Celarent)—No roses are green, All the flowers on my table are roses, ∴ None of the flowers on my table is green.

3. AII (Darii)—All sheep are herbivorous, Some animals are sheep, ∴ Some animals are herbivorous.

4. EIO (Ferio)—No negroes are white, Some Africans are negroes, ∴ Some Africans are not white.

**Valid Moods of the Second Figure.**

We may now proceed to discover the valid moods of the *second figure.* We must first state the *special rules* of the *second figure.* These are: (1) *one premise* must be *negative*; (2) the *major premise* must be *universal.* In the second figure, (whose form is P—M, S—M, S—P), the middle term is the predicate in both the premises, and so, if neither of them is negative, the middle term will remain undistributed, which is contrary to general rule III. Therefore one of the premises must be negative in the second figure. Now if one of the premises is negative, then the conclusion will be negative according to general rule VI, and therefore the major term must be distributed in the conclusion. Therefore it must also be distributed in the major premise (general rule IV). Now the major term is the subject of the major premise in the second figure, and can therefore be distributed only if the premise is universal.

Special rules of the second figure stated and proved.

Which are the valid moods of the second figure? We have already found, while discussing the valid moods of the first figure, that of the sixteen possible combinations of premises, eight may be rejected at once as incapable of yielding any valid conclusion, according to the general rules. The remaining eight are AA, AE, AI, AO, EA, EI, IA, OA. Applying the special rules of the second figure to these eight combinations, we find that only four of them can yield valid conclusions in this figure, *viz.*, AE, AO, EA, EI. From AE—All P is M, No S is M, we may have two conclusions, E and O, *viz.*, No S is P and Some S is not P. Similarly from the combination EA—No P is M, All S is M, we may have two conclusions, E and O, *viz.*, No S is P and Some S is not P. From the combination EI—No P is M,

Determination of the valid moods of this figure.

Some S is M we may draw the conclusion Some S is not P. Similarly the combination AO—All P is M, Some S is not M, yields the conclusion Some S is not P. Thus the *valid* moods of the second figure are six in number, *viz.*, AEE, AEO, EAE, EAO, EIO and AOO. In this figure, as in the first, there are two *weakened* syllogisms or subaltern moods *viz.*, AEO and EAO, because in each case the premises warrant an E proposition as the conclusion, and also because the truth of O follows from the truth of E, O being the subaltern of E. The following mnemonics stand for the four fundamental valid moods of this figure:—Cesare, Camestres, Festino, Baroco. The vowels of these words again stand, as shown, for the propositions of the syllogism.

The special rules of the second figure can be deduced from the dictum de diverso

Just as the special rules of the first figure may be deduced from the dictum of Aristotle, the special rules of the second figure are supposed to rest upon the *'dictum de diverso'*, which may be rendered in English as—"If a certain attribute can be predicated (affirmatively or negatively) of every member of a class, any subject, of which it cannot be so predicated, does not belong to the class". (Mansel). This dictum says that an attribute is predicated of a class, therefore it provides that one of the premises must be universal. It also says that there is to be a subject of which this attribute cannot be predicated. The dictum therefore requires that one of the premises is to be negative.

We may now illustrate the main valid moods of this figure by concrete examples:—

1. EAE (Cesare)—No featherless animals are birds, All swans are birds, ∴ No swans are featherless animals.

2. AEE (Camestres)—All doves are meek, No crows are meek, ∴ No crows are doves.

3. EIO (Festino)—No scarlet flowers are sweet-scented, Some flowers are sweet-scented, ∴ Some flowers are not scarlet flowers.

4. AOO (Baroco)—All warlike men are spirited, Some Indians are not spirited, ∴ Some Indians are not warlike men.

**Valid Moods of the Third Figure**

The special rules of the third figure stated and proved.

We may now state the *special rules of the third figure and prove them.* We must remember that the form of this figure is M—P, M—S, S—P. The special rules of this figure are: (1) the *minor premise* must be *affirmative*; (2) the *conclusion* must be *particular.* If the minor premise is negative, the major premise must be affirmative (rule V), in which case the major term, which is the predicate in the major premise in this figure, cannot be distributed in the major premise. But if the minor premise is negative, the conclusion must be negative (rule VI), and the major term, as its predicate, must be distributed there. But according to rule IV, no term can be distributed in the conclusion without being distributed in the premises. Therefore if the minor premise is negative the fallacy of illicit major is committed. So it must be affirmative. Further, if the minor premise is affirmative, the minor term is not distributed in the minor premise, of which in this figure it is the predicate. It cannot therefore be distributed in the conclusion (rule IV). Therefore the conclusion must be particular.

Valid moods of the third figure.

Let us now see which of the eight combinations of premises, AA, AE, AI, AO, EA, EI, IA, OA, can yield valid conclusions in the third figure. In this figure, AE and AO cannot yield any conclusion, because in each case the minor premise is negative. AA—All M is P, All M is S, yields the conclusion Some S is P. AI—All M is P, Some M is S, also yields the conclusion Some S is P. From EA—No M is P, All M is S, we may draw the conclusion Some S is not P. EI—No M is P, Some M is S, yields the conclusion Some S is not P. IA—Some M is P, All M is S, yields the conclusion Some S is P. OA—Some M is not

P, All M is S, yields the conclusion Some S is not P. The *valid moods* of the *third figure* are therefore AAI, AII, EAO, EIO, IAI and OAO. There is no weakened syllogism or subaltern mood in this figure, since it allows only particular conclusions. The mnemonics that represent the valid moods of this figure are Darapti, Disamis, Datisi, Felapton, Bocardo, Ferison. Here also the vowels of each of the mnemonics represent the propositions of the syllogism.

The special rules of the third figure are supposed to be based upon the *dictum de exemplo,* which is rendered into English as—

The syllogisms of the third figure are said to be based on the dictum de exemplo.

"If anything which is stated to belong to a certain class is affirmed to possess, or to be devoid of, certain attributes, then those attributes may be predicated in like manner of some members of that class". (Welton). This dictum requires that the minor premise should state that something belongs to a class, and therefore it must be affirmative; and also that the conclusion should state that some attributes are to be predicated of some members of that class, which necessitates the conclusion's being particular.

Let us now illustrate the valid moods of this figure by concrete examples:—

1. AAI (Darapti)—Socrates is a Greek, Socrates is wise, ∴ Some wise men are Greeks.

(Here the two premises, though singular, are regarded as universal, because the subject term 'Socrates' carries a definite meaning. Hence the conclusion is formally in the plural number. 'Some' in Logic means 'at least one.')

2. IAI (Disamis)—Some men are honest, All men are mortal, ∴ Some mortals are honest.

3. AII (Datisi)—All birds are winged, Some birds are musical, ∴ Some musical beings are birds.

4. EAO (Felapton)—No negroes are white, All negroes are uncivilised, ∴ Some uncivilised men are not white.

5. OAO (Bocardo)—Some flowers are not red, All flowers are short-lived, ∴ Some short-lived objects are not red.

6. EIO (Ferison)—No men are perfect, Some men are virtuous, ∴ Some virtuous beings are not perfect.

**Valid Moods of the Fourth Figure**

We may now determine the *valid moods* of the *fourth figure.* We must remember that the form of the fourth figure is P—M, M—S, S—P. The *special rules* of the fourth figure are; (1) *if* the *major* is *affirmative* the *minor* must be *universal;* (2) *if either* premise is *negative,* the *major* must be *universal*; (3) if the *minor* is *affirmative,* the *conclusion* must be *particular.* If the major premise is affirmative, then the middle term, which is the predicate of the major premise, will remain undistributed in that premise. It must therefore be distributed in the minor premise (general rule III), and as it is the subject of that premise it can be distributed there only if the minor premise is universal. Thus the first of the above rules is proved. Again, if one of the premises be negative, then the conclusion will be negative (general rule VI), and the major term, which is the predicate in the conclusion, will therefore be distributed there. It must therefore be distributed in the major premise. Now, in the fourth figure the major term is the subject of the major premise, and can be distributed in that premise only if it is universal. Therefore if one of the premises is negative, the major premise must be universal. Thus the second special rule is proved. Again, if the minor premise be affirmative, the minor term, which is the predicate of that premise in the fourth figure, is not distributed in the premise. Therefore it cannot be distributed in the conclusion (general rule IV). Hence the conclusion must be particular; otherwise the

The special rules of the fourth figure stated and explained.

minor term, which is the subject of the conclusion, will be distributed there though it is not distributed in the minor premise. Thus the third special rule is proved.

Valid moods of the fourth figure.

Let us now see which of the eight combinations of premises, AA, AE, AI, AO, EA, EI, IA, OA, yield valid conclusions in the fourth figure. In the fourth figure, AI, AO and OA cannot yield valid conclusions, according to the special rules of this figure. AI and AO are not allowed by the first special rule. OA is not allowed by the second special rule. From AA—All P is M, All M is S, we may have the conclusion Some S is P. From AE—All P is M, No M is S, we may have two conclusions, *viz.*, No S is P and Some S is not P. From IA—Some P is M, All M is S we may draw the conclusion Some S is P. From EA—No P is M, All M is S, we can draw the conclusion Some S is not P. EI—No P is M, Some M is S, yields the conclusion Some S is not P. Thus the *valid* moods of the *fourth* figure are AAI, AEE, AEO, IAI, EAO, and EIO. Of these moods AEO is a *weakened* syllogism or subaltern mood because the premises allow an E proposition as the conclusion, and the truth of O, which is the subaltern of E, follows from the truth of E. If we leave out the subaltern mood, the other valid moods are represented by the mnemonics Bramantip, Camenes, Dimaris, Fesapo, Fresison. The vowels of each of these mnemonics represent the propositions of the syllogisms, as in the other figures.

The special rules of this figure are supposed to follow from Lambert's dictum de reciproco.

The special rules of this figure are supposed to follow from Lambert's *'dictum de reciproco'*, which is rendered in English as—"Whatever has a predicate affirmed, or universally denied, of it, may itself be predicated, particularly and with like quality, of anything which is affirmed of that predicate; and whatever has a predicate universally affirmed of it may itself be universally denied of anything which is universally denied of that predicate." The

following are concrete examples of the main valid moods of this figure:—

1. AAI (Bramantip)—All philosophers are men, All men are mortal, ∴ Some mortals are philosophers.

2. AEE (Camenes)—All roses are sweet-scented, No sweet-scented things are unpleasant, ∴ No unpleasant things are roses.

3. IAI (Dimaris)—Some men are virtuous, All that are virtuous are happy, ∴ Some happy beings are men.

4. EAO (Fesapo)—No politicians are honest, All honest men are trusted, ∴ Some that are trusted are not politicians.

5. EIO (Fresison)—No sheep are carnivorous, Some that are carnivorous are ferocious, ∴ Some that are ferocious are not sheep.

Summary of the valid moods of all the figures

Our examination thus shows that six moods are valid in each figure. In the first figure the valid moods are AAA, AAI, EAE, EAO, AII, EIO. The valid moods of the second figure are EAE, EAO, AEE, AEO, EIO, AOO. Those of the third figure are AAI, IAI, AII, EAO, OAO, EIO. Those of the fourth figure are AAI, AEE, AEO, IAI, EAO and EIO. The above list shows that A can be proved only in one mood and in one figure, *viz.*, in AAA in the first figure. E can be proved in four moods, and in every figure except the third. The moods that prove E are—EAE (fig. 1), EAE, AEE (fig. 2), AEE (fig. 4). I can be proved in seven moods, and in every figure except the second. The moods that prove I are AII and AAI in fig. 1, AAI, IAI and AII in fig. 3, AAI and IAI in fig. 4. O can be proved in twelve moods, and in every figure. The moods that prove O are EAO and EIO in fig. 1 : EAO, AEO, EIO and AOO in fig. 2 : EAO, OAO and EIO in fig. 3 : AEO, EAO and EIO in fig. 4. We have also seen that of the 24 valid moods, five are subaltern moods or weakened syllogisms, viz., AAI, EAO in fig. 1 ; EAO, AEO in fig. 2 ; and AEO in fig. 4. Many logicians do not mention these moods when they draw up a list of valid moods of all the figures.

If we ignore these weakened syllogisms or subaltern moods, the fundamental valid moods in all the figures are 19 in number. These are all mentioned in the following mnemonic lines:

*Barbara, Celarent, Darii, Ferio*que prioris:[1]
*Cesare, Camestres, Festino, Baroco* secundae:[1]
Tertia[1] *Darapti, Disamis, Datisi, Felapton,*
*Bocardo, Ferison* habet:[1] quarta insuper addit[1]
*Bramantip, Camenes, Dimaris, Fesapo, Fresison.*

"If in a syllogism the same conclusion can still be obtained although for one of the premises we substitute its subaltern, the syllogism is said to be a *strengthened* syllogism" (Keynes). In a strengthened syllogism one of the premises is unnecessarily strengthened, that is, it can be replaced by its subaltern without affecting the conclusion. Thus Darapti—All M is P, All M is S, ∴ Some S is P, is a strengthened syllogism, because the same conclusion can be obtained even if one of the premises is replaced by its subaltern. In three moods, *viz.*, AAI in fig. 3, and EAO in figs. 3 and 4 the middle term is twice distributed, which is not demanded by any of the rules of the syllogism. Similarly Bramantip—All P is M, All M is S, ∴ Some S is P—is a strengthened syllogism. In this case the major term is undistributed in the conclusion, while it is distributed in the major premise; and hence for the major premise we may substitute its subaltern, which is an I proposition, and still obtain the same conclusion. Every syllogism with universal premises and a particular conclusion is strengthened, with the single exception of AEO in the fourth figure. Therefore all the weakened syllogisms except AEO in the fourth figure are, at the same time,

**Strengthened and weakened syllogism explained.**

[1] **Prioris = 'of the first' (figure); secundae = 'of the second' (figure): tertia...habet = 'the third (figure) includes' (the moods mentioned in between the two words); 'quarta insuper addit' = 'the fourth (figure) adds over and above'.**

strengthened ones, and in each of these cases a particular conclusion follows from two universal premises. Thus we have a list of eight strengthened syllogisms, *viz.*, AAI and EAO in fig. 1; EAO, AEO in fig. 2; AAI, EAO in fig. 3; AAI and EAO in fig. 4. We know that four of these are subaltern moods or weakened syllogisms, *viz.*, AAI, EAO in fig. 1; EAO, AEO in fig. 2. In addition to these four, there is another weakened syllogism, *viz.*, AEO in the fourth figure. Since in each of the subaltern moods we can, instead of a particular conclusion, put its subaltern universal as the conclusion, these moods are supposed to be worthless and misleading. But if we are to mention the possible valid moods of all the figures, we should not omit subaltern moods, and should name all the 24 possible valid moods. We have already pointed out that if we omit the weakened syllogisms of all the figures, we are left with nineteen valid moods in all. If we omit all the strengthened and weakened syllogisms of all the figures, we are left with only fifteen valid moods.

The first figure the most natural.

Aristotle, as we have already remarked, regarded the first figure as the perfect figure. It is undeniable that the first figure is the most natural, and by far the most important of all the figures. In it, the subject of the conclusion is also the subject of the minor premise, and the predicate of the conclusion is the predicate of the major premise. Thus the meaning of S and P, the minor and the major terms, is the same in the premises and in the conclusion. If S is read in denotation in the minor premise, it may be read in denotation in the conclusion; and if P is read in connotation in the major premise, it can be read in connotation in the conclusion. Further, it is natural that the predicate of a proposition should have greater extension than the subject. "The heavier term," says Bosanquet, "is naturally the predicate of a proposition." 'All men are mortal' is a more natural proposition than 'Some mortals are men.' In the first figure this condition is fulfilled, and the predicate in each of the propositions of a syllogism

in this figure has usually greater extension than the subject. Furthermore, in this figure alone the major premise states the principle and the minor premise gives an instance of it. This is the only figure which can prove A, E, I, O, that is, all kinds of propositions. Besides, from the scientific point of view an A proposition is the most important, since by means of it we can state a law, and the first figure is the only figure which can prove an A proposition.

The uses of each of the four figures of the syllogism.

Though the first figure is the most important, some arguments naturally fall in other figures. The second figure proves only negative conclusions, and is called the exclusive figure. While the *first* figure is useful for *discovery* and *proof*, the *second* is useful for the purposes of *disproof*, because it *proves negatives*. By means of it we may go on excluding various suppositions as to the nature of the object under investigation, whose real character we wish to ascertain. Such an argument as, 'Such and such orders have such and such properties, This plant has not those properties, ∴ It does not belong to that order,' falls naturally in the second figure. The *third* figure *proves* only *particulars*. By means of it we can *disprove universal* propositions. By proving O we can disprove A, and by proving I we can disprove E. It is a natural figure when the subject of the premises is a singular term, *e.g.*, Socrates is a Greek, Socrates is a philosopher, ∴ Some philosophers are Greeks. The fourth figure is supposed to be unnatural, but arguments can be proved in this figure as in any other figure. The *fourth* figure is said to be useful for *discovery* or *exclusion* of the different species of a genus. Lambert sums up the utility of the different figures in the following words:—"The first figure is suited to the discovery or proof of the properties of a thing; the second to the discovery or proof of the distinctions between things; the third to the discovery or proof of instances and exceptions; the fourth to the discovery or exclusion of the different species of a genus."

The fourth figure was not recognised by Aristotle, on the ground that it was unnatural. We have already seen that this figure can prove its conclusions as faultlessly as any other figure. However it is very seldom that arguments fall naturally in this figure. This figure is the reverse of the first figure. If we transpose the premises of the first figure, we get the form of the fourth figure as determined by the position of the middle term in relation to the extremes. The form of the premises of the first figure is M—P, S—M ; transposing these, we get S—M, M—P, which is of the same form as the fourth figure. If this is so, then if from a mood of the first figure we draw P—S as the conclusion instead of S—P, by transposing its premises, then we have a mood in the fourth figure. Aristotle held that some moods in the first figure allow us to draw P—S as the conclusion. If we do so, the conclusion is indirect, and such a mood he calls an indirect mood. Let us now see what moods may yield an indirect conclusion in the first figure.

Should the fourth figure be recognised as a distinct figure?

From 'All men are mortal, All philosophers are men,' we may legitimately draw the indirect conclusion, 'Some mortals are philosophers.' If we do so, the mood becomes Bramantip in the fourth figure. Similarly, from 'No men are perfect, All philosophers are men,' we may draw the indirect conclusion 'No perfect beings are philosophers'. Here we have the mood Camenes in the fourth figure. Again, from the premises 'All roses are sweet-scented, Some flowers are roses', we may draw the indirect conclusion, 'Some sweet-scented things are flowers.' In this case we have the mood Dimaris in the fourth figure. From the premises 'All virtuous men are trusted, No politicians are virtuous', we may draw the indirect conclusion, 'Some that are trusted are not politicians.' In this case we have the mood Fesapo in the fourth figure. Lastly, from the premises 'Some flowers are white, No oranges are flowers,' we may draw the indirect conclusion, 'Some white things are not oranges.' This is the mood Fresison in the fourth figure. Thus by drawing indirect conclusions from valid and invalid moods of the first figure, Aristotle provided all the moods of the fourth figure, *viz.*, Bramantip, Camenes, Diamaris, Fesapo, Fresion. But if we recognise indirect moods of the first figure, there is no harm in recognising a fourth figure which can directly yield the same conclusions.

Johnson points out that the fourth figure cannot be proved to be unnatural. Some arguments do appear in this figure. He points out that the worthlessness of the fourth figure is ordinarily proved by the argument—"Any argument worthy of logical recognition must be such as would occur in ordinary discourse. Now it will be found that no argument occuring in ordinary discourse is in the fourth figure. Hence. no argument in the fourth figure is worthy of logical recognition". But this very argument, he points out, is in the fourth figure. So it cannot be established that the fourth

figure is useless. The following example given by Keynes falls naturally in the fourth figure: None of the Apostles were Greeks, Some Greeks are worthy of all honour, ∴ Some worthy of all honour are not Apostles. If it be said that the fourth figure is nothing but the first with a converted conclusion, the remark does not apply to Fesapo and Fresison, for there is no valid mood of the first figure, the conversion of whose conclusion can provide the conclusions required by them. Besides, if we define figure as the form of the syllogism as determined by the position of the middle term, then the place of the fourth figure cannot be denied.

## Pure Hypothetical and Disjunctive Syllogisms

Having ascertained the nature of pure categorical syllogisms, we may now discuss briefly pure *hypothetical* and *disjunctive syllogisms.* The same principles which apply to pure categorical syllogisms, apply to these syllogisms as well, and no new principles are involved. A distinction is often drawn between *hypothetical* and *conditional* propositions. The form of a hypothetical proposition is If P then Q, while that of a conditional proposition is If S is M, it is P. In a *conditional* proposition there is a *common element between* the *antecedent* and the *consequent,* while there is *no* such *common element* between them in a *hypothetical* proposition. But since the same principles are involved in both hypothetical and conditional syllogisms, we need not treat of them separately. The *hypothetical* proposition is *abstract* in form, while the *conditional* proposition *may be concrete.* Further we have previously remarked that the traditional fourfold distinction of propositions applies clearly to hypothetical propositions in their conditional or denotative form, which is concrete.

Ignoring the distinction between hypothetical and conditional propositions, we may define a pure *hypothetical syllogism* as a reasoning consisting of *two hypothetical premises* and a *hypothetical conclusion.* We may now illustrate some valid moods by means of hypothetical syllogisms. Barbara—If any S is M, always that S is P; If any S is Q, always that S is M; therefore If any S is Q, always that S is P. The

Pure Hypothetical Syllogisms explained and illustrated.

following is a concrete example: If any person is cruel, always he is hated; If any person recklessly wounds the feelings of another, always he is cruel; therefore If any person recklessly wounds the feelings of another, always he is hated. Cesare—If any S is M, never that S is P; If any S is Q, then always that S is P; therefore If any S is Q, never that S is M. If any man is foolish, he is never respected; If any man is selfless, he is always respected; therefore If any man is selfless he is never foolish. Bocardo—If an S is M, then sometimes not that S is P; If any S is M, then always that S is Q; therefore If an S is Q, then sometimes not that S is P. If a state is weak, sometimes it is not feared; If any state is weak, always it fails to preserve its independence; therefore If a state fails to preserve its independence, sometimes it is not feared. Bramantip—If any S is M, always it is P; If any S is P, always it is Q; therefore If an S is Q, sometimes it is M. If any currency is inflated, always it uses inconvertible paper money; If any currency uses inconvertible paper money, always it loses respect in the foreign market; therefore If a currency loses respect in the foreign market, it is sometimes inflated. We need not multiply more examples of hypothetical syllogisms; the above are sufficient to exhibit their nature. The antecedent and the consequent of a hypothetical proposition are regarded as similar to the subject and the predicate of a categorical proposition when it forms part of an inference.

We know that the force of any syllogistic argument depends upon the necessity with which the conclusion follows from the premises. This necessity can be expressed by both hypothetical and categorical syllogisms. So conditional propositions can be reduced to categorical ones. Thus the argument. if any man is cruel, he is always hated; If any man is cruel, he is always selfish; therefore If a man is selfish he is sometimes cruel, can be reduced to the categorical syllogism, All cruel men are hated, All cruel men are selfish, therefore All selfish men are hated. We must remember that Barbara is the only form of the hypothetical syllogism

which is of much importance, since hypothetical propositions are by nature universal and abstract.

The possibility of the disjunctive syllogism considered.

Logicians have not generally considered the possibility of disjunctive syllogisms, but in some cases such syllogisms are undoubtedly possible. Since they can never be negative, the only disjunctive syllogisms possible are those in which the conclusion and premises are affirmative propositions. Barbara is the only form of the disjunctive syllogism that is of any importance. In a disjunctive syllogism we can secure a middle term only when the minor premise negatives one of the alternatives of the major. Thus the following disjunctive syllogism is in the mood Barbara:—S is either P or Q, S is either P′ or R, ∴ S is either Q or R. The argument will be apparent if we reduce it to the hypothetical form: If S is P′ it is Q, If S is R′ it is P′, ∴ If S is R′, it is Q. The following argument is *not* a disjunctive syllogism: S is either P or Q, S is either P or R, ∴ S is either P or Q or R.

### Reduction of Syllogisms

"By *reduction* is meant a process whereby the reasoning contained in a given syllogism is expressed in some other mood or figure. Unless an explicit statement is made to the contrary, reduction is supposed to be to figure 1" (Keynes).

Reduction regarded as indispensable by Aristotle

Aristotle, we have already stated, believed that the dictum de omni et nullo is the ground of all syllogistic inference. But this dictum, which asserts that whatever can be affirmed or denied of a class distributively can be, in the same way, affirmed or denied of anything belonging to that class, applies directly to the first figure only, the empty scheme of which is M—P, S—M, S—P. We have seen that the dictum requires that the major premise should be universal and the minor premise affirmative, and that the conclusion should be affirmative if the

major premise is affirmative and negative if it is negative. We have also pointed out that the general rules of the syllogism can be deduced from the dictum by generalising it. Aristotle and his followers however thought that since the dictum applies directly to the first figure only, this is the perfect figure, and the moods of other figures should be reduced to the moods of the first figure in order to establish their validity. So the doctrine of reduction has come to be a part of the syllogistic doctrine.

**Direct and indirect reduction explained.**

Reduction may be either *direct* or *indirect*. In the case of *direct* reduction a mood is changed into some other mood with the help of the processes of *immediate inference*. Thus Cesare can be converted to Celarent by converting the major premise. The syllogism No P is M, All S is M, ∴ No S is P is in the mood Cesare; by converting its major premise we get the syllogism No M is P, All S is M, ∴ No S is P, which is in the mood Celarent. Aristotle reduced Baroco and Bocardo indirectly, because these moods cannot be reduced to the moods of the first figure without the introduction of negative terms, that is, without taking the help of obversion. But Aristotle was averse to the introduction of negative terms. So he reduced them indirectly. The *indirect* method of proof was frequently adopted by Euclid in his Elements of Geometry. Indirect reduction is an effective weapon in controversy, and is perhaps one of the methods most commonly employed for that purpose.

**Indirect reduction of Baroco.**

Let us see how Baroco and Bocardo can be reduced indirectly to the first figure. Take Baroco, which is All P is M, Some S is not M, ∴ Some S is not P. Suppose this argument is not valid. Now, the truth of the premises of the argument cannot be questioned, since it is a syllogism, the premises of which are always accepted as true. Thus the falsity of the argument must be due to the falsity of the conclusion. If the conclusion Some S is not P is not true, then according to the law of excluded

middle its contradictory, All S is P, must be true. Now, if this is so, then the propositions All P is M, Some S is not M, All S is P are true together. Now combining All S is P with the original major premise, we get the syllogism, All P is M, All S is P, ∴ All S is M. This argument is in the mood Barbara, and therefore its validity cannot be questioned. But if the conclusion established here, *viz.*, All S is M, is true, then its contradictory, Some S is not M, must be false, according to the principle of contradiction. But Some S is not M cannot be false, because it is the minor premise of the original syllogism and has been given as true. Therefore the new minor premise, All S is P, must be false, and its contradictory, Some S is not P, true. Therefore the original argument, All P is M, Some S is not M, ∴ Some S is not P, is a valid argument. Such an indirect reduction is called *Reductio ad impossible*, or Reductio per impossible, Deductio ad impossible or Deductio *ad absurdum*.

Indirect reduction of Bocardo.

We may now reduce Bocardo indirectly to figure I. The argument is, Some M is not P, All M is S, ∴ Some S is not P. If this argument is not true, then the conclusion must be false, since the truth of the premises cannot be questioned. If the given conclusion is false, then its contradictory, All S is P, must be true, according to the principle of excluded middle. Then the propositions Some M is not P, All M is S, All S is P are true together. Combining All S is P with the original minor premise we get the syllogism, All S is P, All M is S, ∴ All M is P. Here the process of reasoning is perfect, the argument being in Barbara. But if the new conclusion is true, then its contradictory, Some M is not P, must be false, according to the law of contradiction. But Some M is not P cannot be false, since it is the major premise of the original syllogism. Therefore the new major premise, All S is P, must be false, and its contradictory, Some S is not P, the conclusion of the original syllogism, true. Therefore the original syllogism, Some M is not P, All M is S, ∴ Some S is not P, is valid.

We have already referred to certain mnemonics that have been adopted by logicians to name the valid moods of the different figures. They are—

BARBARA, CELARENT, DARII, FERIO (FIG. 1);
CESARE, CAMESTRES, FESTINO, BAROCO (FIG. 2);
DARAPTI, DISAMIS, DATISI, FELAPTON, BOCARDO, FERISON (FIG. 3);
BRAMANTIP, CAMENES, DIMARIS, FESAPO, FRESISON (FIG. 4).

This list omits the five subaltern moods, and names the nineteen fundamental valid moods of different figures. De Morgan calls the mnemonic lines magic words, full of meaning, and by these words valid moods of different figures have been denoted for many centuries. The only meaningless letters of the mnemonic words are *b* (not initial), *d* (not initial), *l*, *n*, *r*, *t*. The vowels of each word give the quality and quantity of the propositions of which the syllogism is composed. The initial letters in the case of figures 2, 3 and 4 show to which of the moods of figure 1 the given mood is to be reduced. It is to be reduced to that mood of the first figure which has the same initial letter. Thus Camestres is reduced to Celarent, Darapti to Darii, Fesapo to Ferio, Bramantip to Barbara and so on. *s* (in the middle of a word) indicates that in the process of reduction the preceding proposition is to be simply converted, *s* (at the end of a word) shows that the conclusion of the new syllogism has to be simply converted in order that the given conclusion may be obtained. *p* (in the middle of a word) signifies that the preceding proposition is to be converted per accidens, *e.g.*, in the reduction of Darapti to Darii. *p* (at the end of a word) implies that the conclusion obtained by reduction is to be converted per accidens to get back to the original conclusion. *m* indicates that in reduction the premises have to be transposed (metathesis præmissarum), as in the case of Camestres or Bramantip. *c* signifies that the mood is to be reduced indirectly (*i.e.*, by reductio per impossibile, as shown before). The position

The meaning of the letters of the mnemonic words.

of the letter *c* indicates that we are to omit the premise preceding it, that is, the other premise is to be combined with the contradictory of the conclusion of the original syllogism.

We have already considered how Baroco and Bocardo were reduced indirectly, but logicians have reduced them directly as well. In the case of direct reduction, Faksoko is sometimes given as a mnemonic for Baroco, and Doksamosk for Bocardo. Here *k* denotes obversion, *ks* obversion followed by conversion, that is, contraposition, and *sk* conversion followed by obversion. Let us now reduce Baroco and Bocardo directly.

Direct reduction of Baroco and Bocardo.

The mood Baroco (Faksoko)* may be represented by the syllogism All P is M, Some S is not M, therefore Some S is not P. Now contraposing the major premise of the syllogism and obverting the minor premise, we get the following syllogism: No not-M is P, Some S is not-M, therefore Some S is not P, which is in the mood Ferio. Similarly the mood Bocardo (Doksamosk) —Some M is not P, All M is S, ∴ Some S is not P—can be reduced to Darii by contraposing the major premise of the syllogism and transposing the premises. The syllogism will then stand as All M is S, Some not-P is M, ∴ Some not-P is S. Now by first converting this conclusion and then obverting, as required by *sk* at the end of Doksamosk, we get the original conclusion, Some S is not P.

We may now *directly reduce* some of the moods of figures 2, 3 and 4 to the moods of the first figure. Camestres—All P is M, No S is M, ∴ No S is P—can be reduced to Celarent by simply converting the minor premise and transposing the premises. When thus reduced the syllogism will become No M is S, All P is M, ∴ No P is S. Now simply converting the new conclusion, as required by *s* at the end of Camestres, we get the original conclusion, No S is P. Darapti—All M is P, All M is S, ∴ Some S is P—can be reduced to Darii by converting the minor premise by limitation. The syllogism

Direct reduction of 2nd, 3rd and 4th figures.

will then become All M is P, Some S is M, ∴ Some S is P. Disamis—Some M is P, All M is S, ∴ Some S is P—may be reduced to the syllogism All M is S, Some P is M, ∴ Some P is S, which is in the mood Darii, by simply converting the major premise of Disamis and transposing the premises. The new conclusion, Some P is S, can be simply converted to Some S is P, as required by *s* at the end of Disamis. Bramantip—All P is M, All M is S, ∴ Some S is P—can be reduced to Barbara by transposing the premises. The syllogism will then become All M is S, All P is M, ∴ All P is S. By converting this new conclusion per accidens, as required by *p* at the end of Bramantip, we get the original conclusion, Some S is P. Fesapo—No P is M, All M is S, ∴ Some S is not P—can be reduced to Ferio by converting the minor premise per accidens and the major premise simply. The syllogism will then become No M is P, Some S is M, ∴ Some S is not P. We need not reduce other moods directly to the moods of the first figure, the above examples being sufficient to show the principles involved.

Though Aristotle reduced only Baroco and Bocardo indirectly, other moods of figures 2, 3 and 4 may also be reduced *indirectly* to the moods of the first figure. We may here give only one example by way of illustration. Let us take Darapti—All M is P, All M is S, ∴ Some S is P. If the conclusion, Some S is P, is not true, then its contradictory, No S is P, must be true. Then the propositions All M is P, All M is S, and No S is P will be true together. Now combining No S is P with the original minor premise we get the syllogism No S is P, All M is S, ∴ No M is P. This argument is in the perfect mood Celarent. If the new conclusion is valid, then its contrary, All M is P, becomes invalid, because we know that of two contrary premises if one is true, the other is false according to the principle of contradiction. But All M is P, which is the major premise of the original syllogism, cannot be false. There is no fallacy in the new argument, it being in the perfect mood Celarent. The new

Indirect reduction of Darapti.

minor premise, which is the original minor premise, cannot be false. Therefore the new major premise, No S is P, which is the contradictory of the conclusion of the original syllogism, must be false, and therefore the original conclusion, Some S is P, must be true. Therefore the original syllogism, All M is P, All M is S, ∴ Some S is P, is true.

A mood of a figure may be reduced to another mood of that figure.

We may also point out that a mood of a figure may be reduced to another mood of the same figure. Thus Barbara, All M is P, All S is M, ∴ All S is P, may be reduced to Celarent by the obversion of the major premise and of the conclusion. The syllogism will then become No M is not-P, All S is M, ∴ No S is not-P. Similarly Celarent may be reduced to Barbara by obverting the major premise and the conclusion. Darii may be reduced to Ferio and Ferio to Darii by obverting the major premise and the conclusion. Barbara and Darii are reducible to each other by means of the indirect method. We need not however show how this is possible.

Should the problem of reduction be treated of in connection with the doctrine of the syllogism?

Whately, following Aristotle, argues that since the figures 2, 3 and 4 are imperfect, the moods of these figures can be proved valid by reducing them to the moods of the first figure. So according to him we cannot but consider reduction in the treatment of the syllogism. Fowler holds that though the validity of the moods of figures 2, 3, and 4 can be proved by other dicta which are as self-evident as the dictum of Aristotle, yet if we reduce them to moods of the first figure we can be sure of their validity. Ueberweg appears to be right when he points out that to prove the validity of the moods of syllogisms we do not require the help of any axiom whatsoever. The validity of particular moods is as self-evident as the validity of the axioms themselves. It is undeniable that the axioms are deductions from the nature of

syllogistic reasoning itself, and its validity does not depend upon them. It is also true that if we depend upon axioms to prove the validity of particular pieces of syllogistic reasoning, we need not depend upon the axiom of Aristotle alone. Every figure has its independent self-evident axiom, as we have already shown. We have also seen that some arguments fall naturally in figures 2, 3 and 4 instead of figure 1. In spite of all this, it appears that the treatment of reduction as a part of the doctrine of the syllogism is useful, inasmuch as it shows the equivalence between the moods in different figures, and thus reminds us that syllogistic reasoning is a whole, a unity, though it admits of being separated into parts, or plurality. Further, reduction is an excellent logical exercise. In conclusion we may remark that, though reduction is useful, yet it is not indispensable because the validity of moods of different figures can be proved with the help of general rules and special rules of different figures.

Following the principles of reduction pure hypothetical syllogisms of imperfect figures can be reduced to the moods of the first figure which is regarded by Aristotle to be the perfect figure. But we need not reduce hypothetical syllogisms of imperfect figures to the moods of the perfect figure, as such reduction does not involve any new principle.

## CHAPTER XII

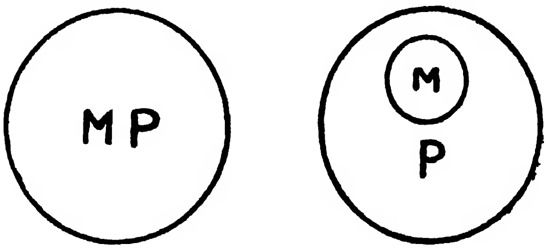
### DIAGRAMMATIC REPRESENTATION OF SYLLOGISMS

Euler's Diagrams may be used to illustrate the various forms of syllogistic reasoning. The method of procedure will be easy to grasp if we first apply them to a syllogism in Barbara,—

All M is P,
All S is M,
∴ All S is P.

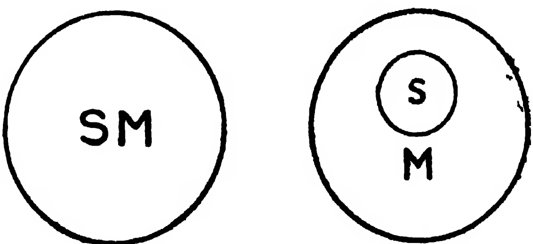
First the premises must be separately represented by means of appropriate diagrams. In this syllogism each premise is an A proposition, and this requires two diagrams to represent it. Thus we have—

M a P,




I II

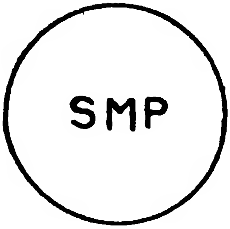
S a M,



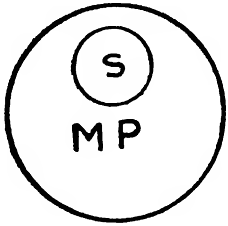

(a) (b)

To obtain the conclusion, each of the cases yielded by the major premise must be combined with each of those yielded by the minor. Thus we have the following diagrams—

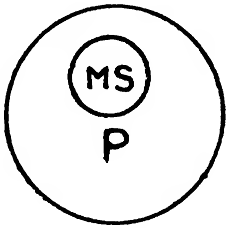
I and (a) give—



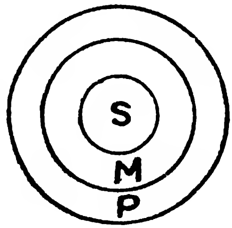

I and (b) give—




II. and (a) give—




II. and (b) give—



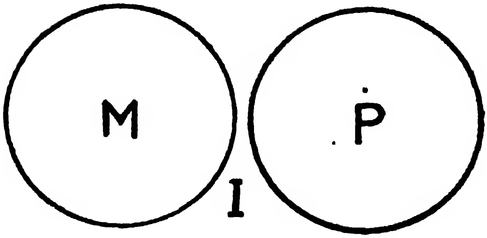

To find the conclusion, we have to ask what relation between S and P is common to all these four diagrams. The answer plainly is that in the first of them [I (a)] S and P are co-extensive, and in the other three the class S is entirely included in the class P. In all four cases, therefore, All S is P. We proceed to illustrate the other three valid moods of the first figure.

*Celarent*,—

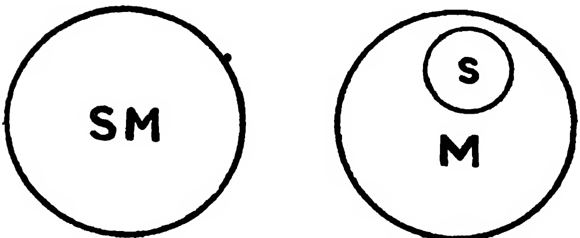
No M is P,
All S is M,
∴ No S is P.

Here the major premise requires only one diagram, the minor, two. Thus,—

M e P,



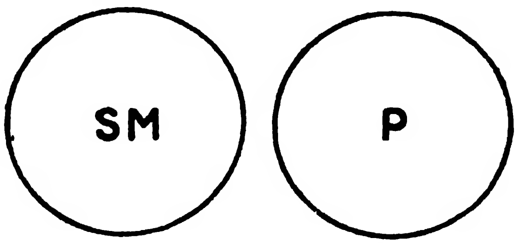

S a M



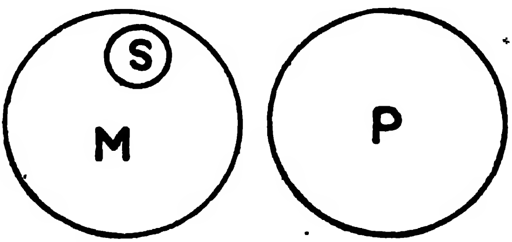

(a) (b)

In combination,

I. and (a) give—




I. and (b) give—



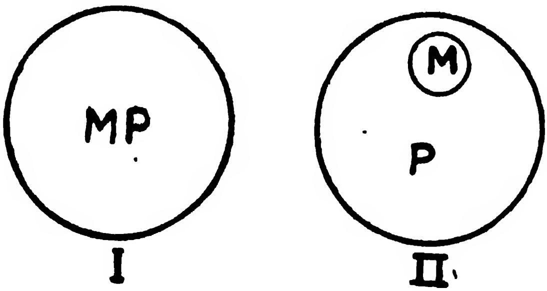
Asking now what the relation between S and P is, we see that in both cases the whole of S is excluded from the whole of P; *i.e.*, No S is P.

*Darii.*—

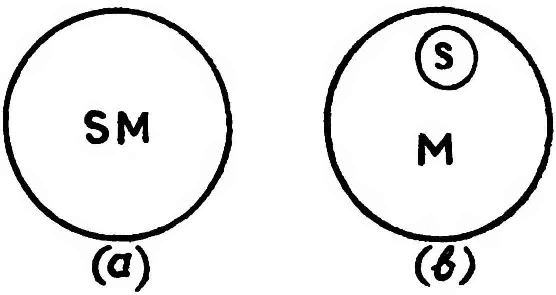
All M is P,
Some S is M,
∴ Some S is P.

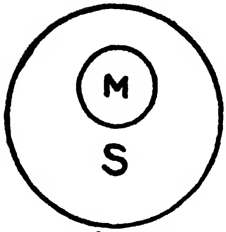
Here the major premise requires two diagrams, the minor, being an I proposition, as many as four. Thus,—

M a P,

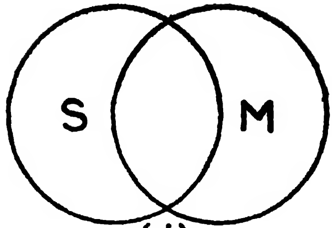



S i M,





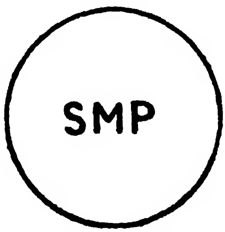

(c)



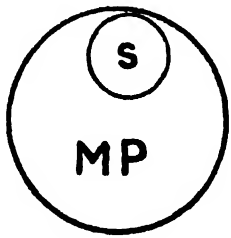

(d)

When we combine these, we find that II and (c) themselves give four alternative diagrams, and II and (d) two alternatives. The total combinations are as follows:—

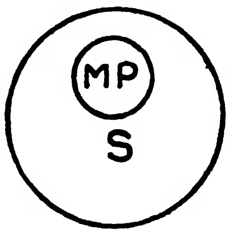
I. and (a) give—



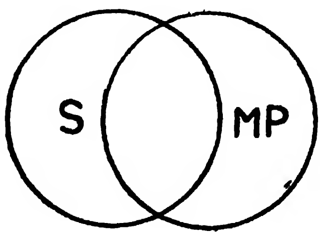

I. and (b) give—



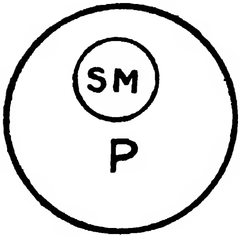

I. and (c) give—



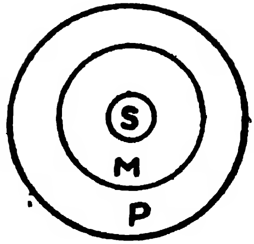

I. and (d) give—



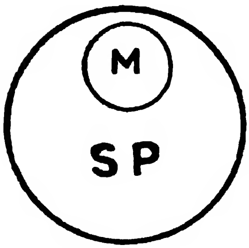
II and (a) give—



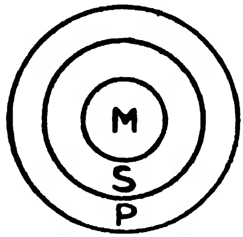

II. and (b) give—



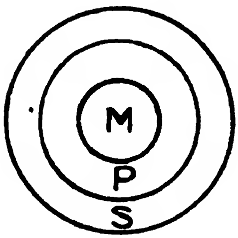

II. and (c) give—



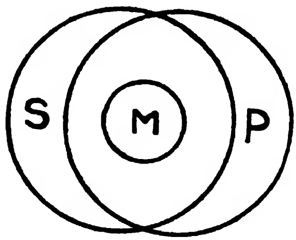

or



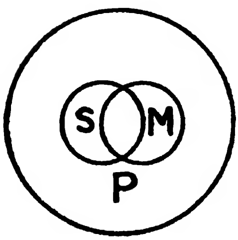

or



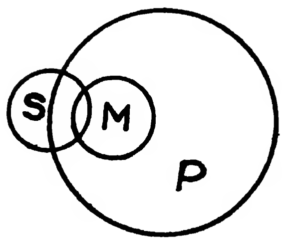

or



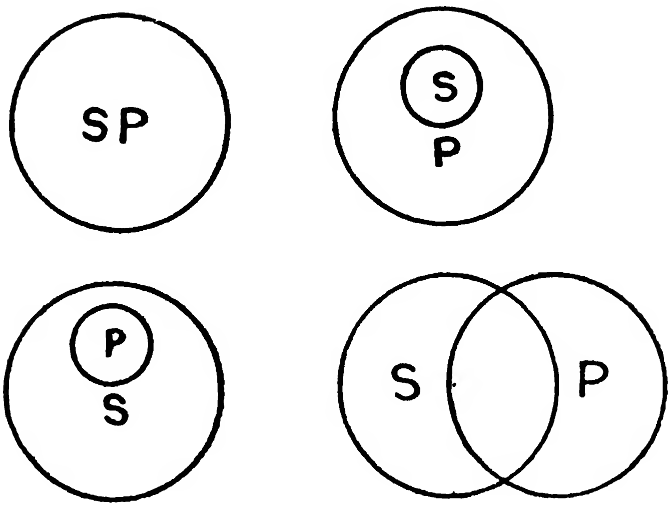

II. and (d) give—




or



To find the conclusion, if we leave M out of account, we find that the above diagrams reduce to the following four, *viz.*,—



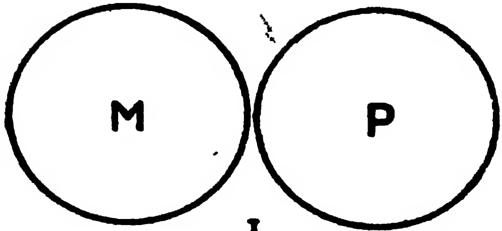

and what is common to *all* these four cases is that some members of the class S coincide with some members of the class P. That is, Some S is P.

*Ferio.*—

No M is P,
Some S is M,
Some S is not P.

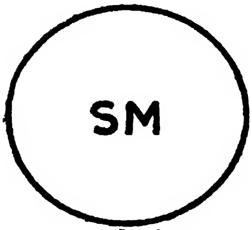
Here we have—

M e P,

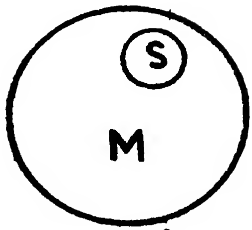



I

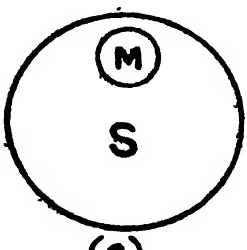
S i M,



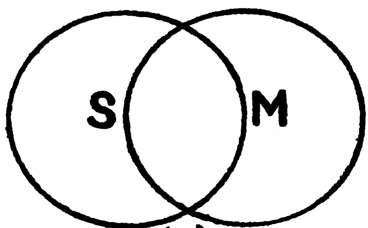

(a)




(b)



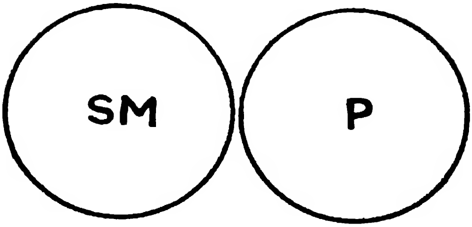

(c)



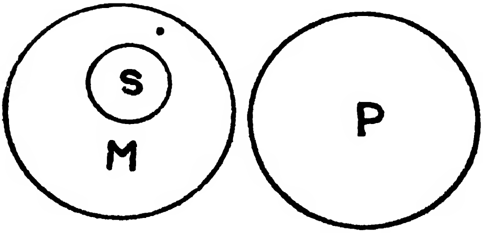

(d)

This gives the following combinations:—

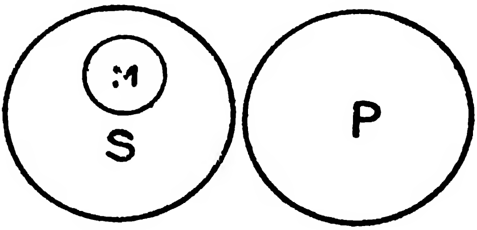
I. and (a) give—



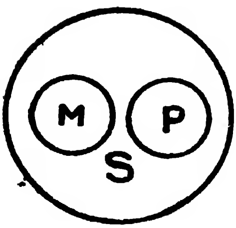

I. and (b) give—

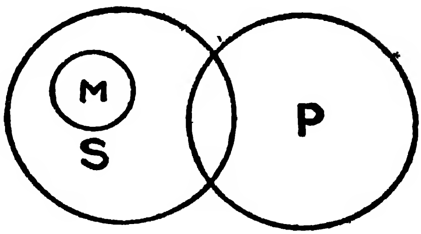



I. and (c) give—

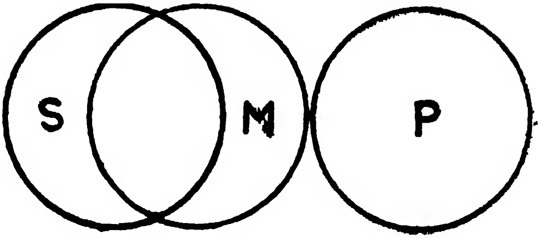



or

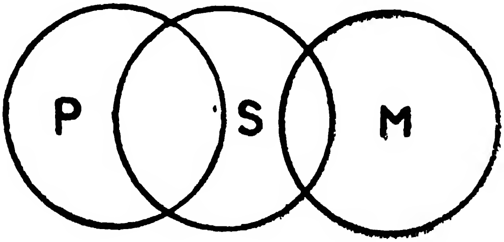


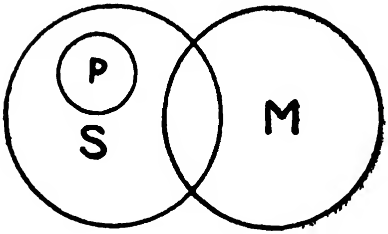
I. and (d) give—



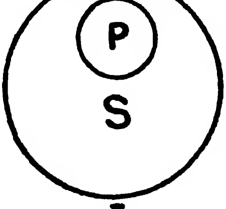

or



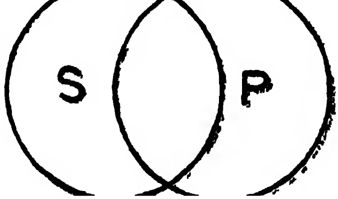

or



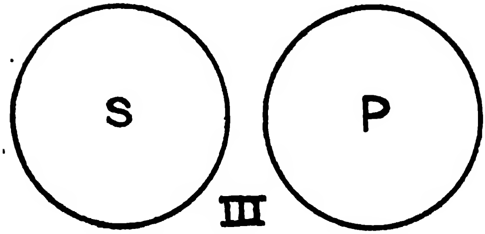

If we now leave M out of account, we find that the above diagrams reduce to the following three, *viz.*—




I




II




and what is common to *all* these three cases is that some members of the class S are excluded from the class P *i.e.*, Some S is not P.

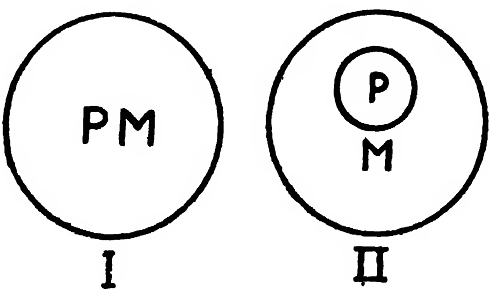
We also give an example of one mood in each of the remaining figures.

In the second figure take *Baroco*,—

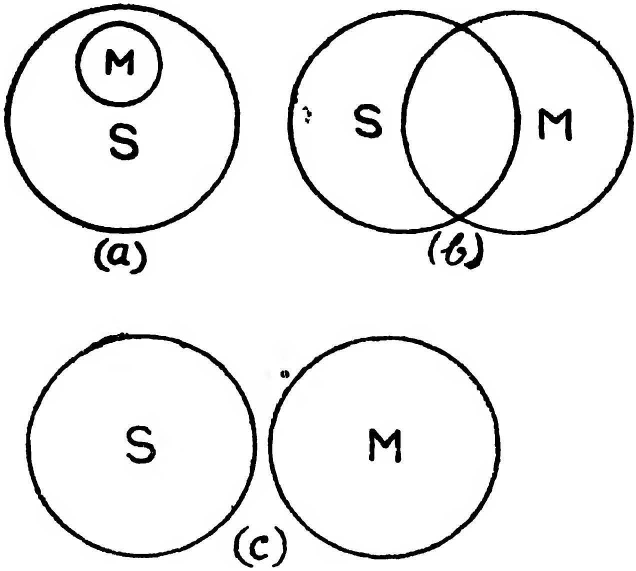
All P is M,
Some S is not M,
∴ Some S is not P.

Here we have-

P a M,

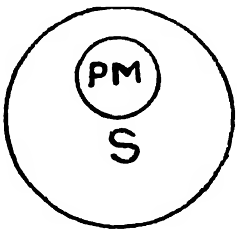


S o M,

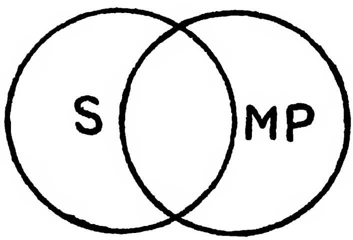



Then,

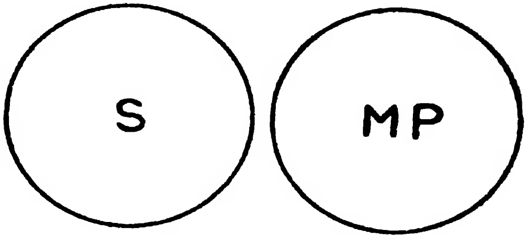
I. and (a) give—



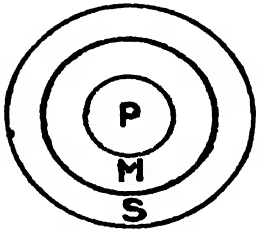

I. and (b) give—



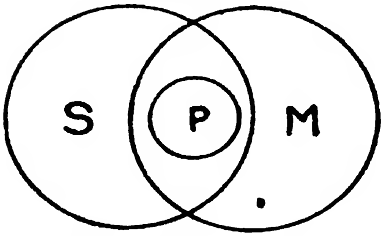
I. and (c) give—



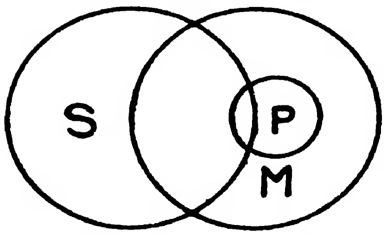

II. and (a) give—



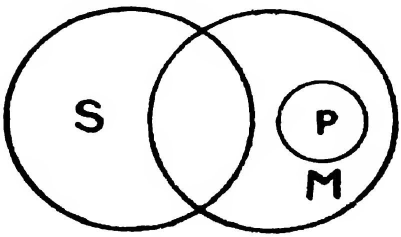

II. and (b) give—



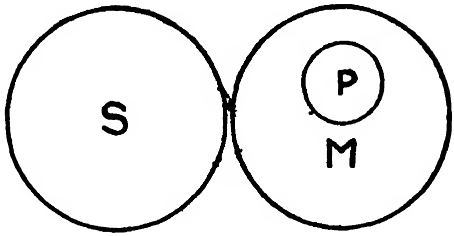

or



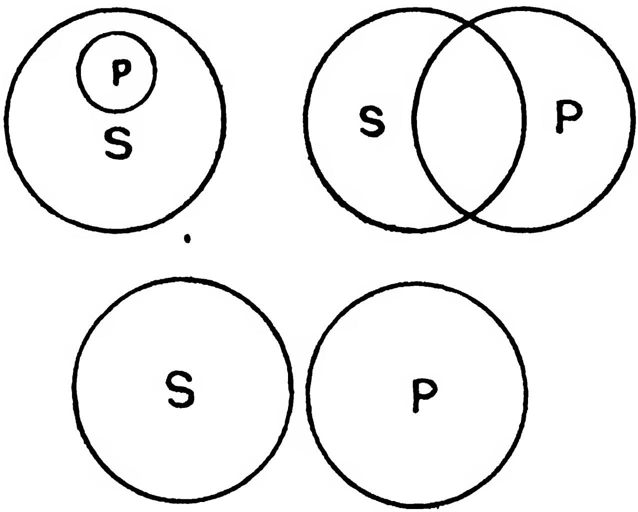

or



II. and (c) give—




If we now leave M out of account, we find that the relations between S and P in all these diagrams are expressed in the three figures—



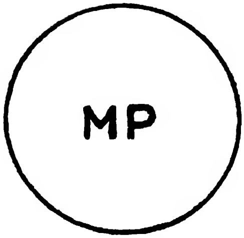

That is, the conclusion is, Some S is not P.

In the third figure, take *Disamis*—

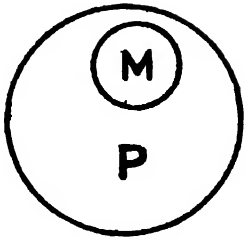
Some M is P,
All M is S,
∴ Some S is P.

Here we have,

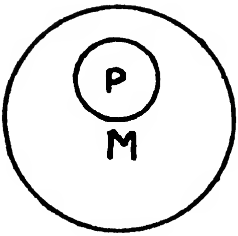
M i P,



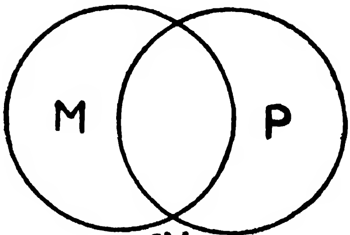

I




II

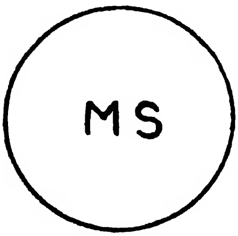



III

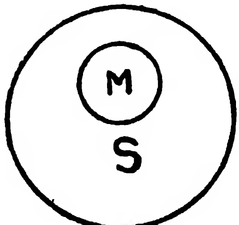



IV

M a S,



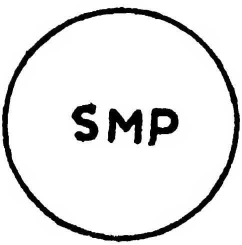

(a)



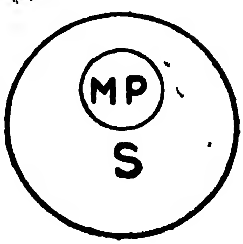

(b)

Then,

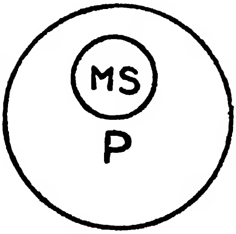
I. and (a) give—



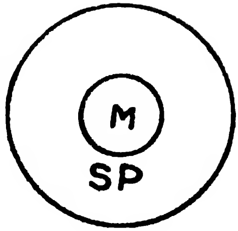

I. and (b) give—



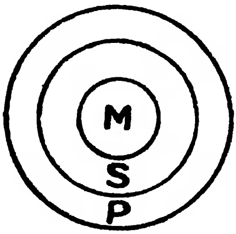
II. and (a) give—



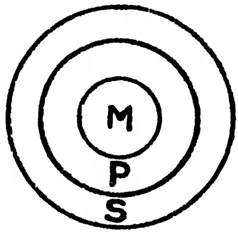

II. and (b) give—



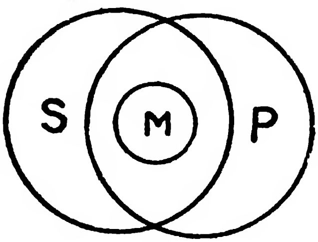

or



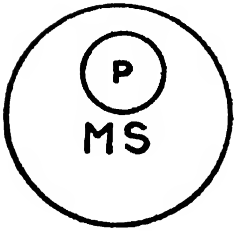

or



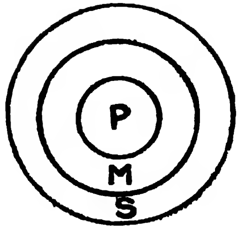

or



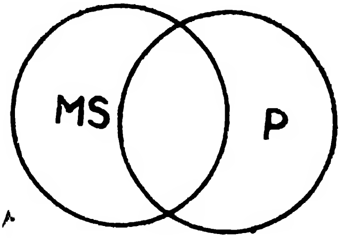

III. and (a) give—



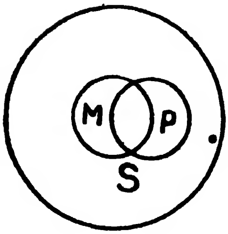

III. and (b) give—



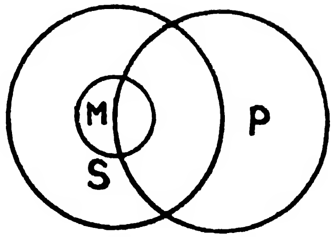
IV. and
(a) give—



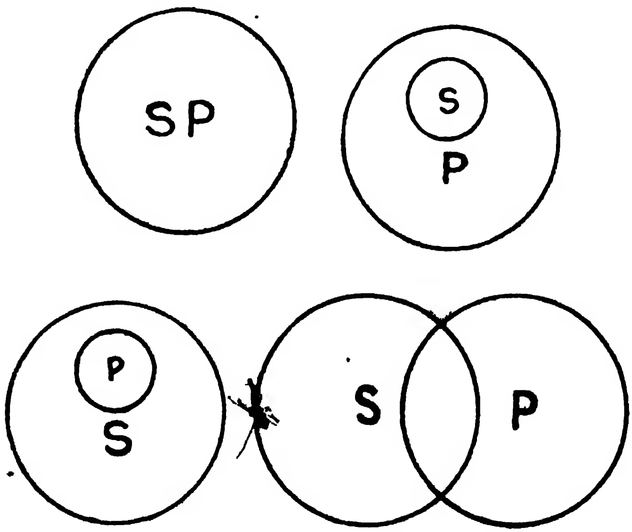

IV. and
(b) give—




or




If we neglect M, the above diagrams reduce to four only, *viz.*,—



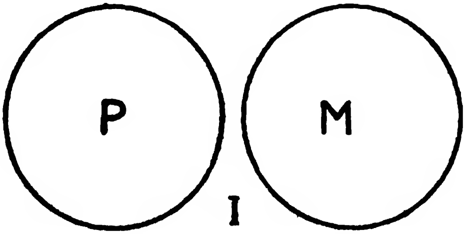
and what is common to all four is that some members of the class S are included in the class P. That is, Some S is P.

In the fourth figure, take *Fresison*,

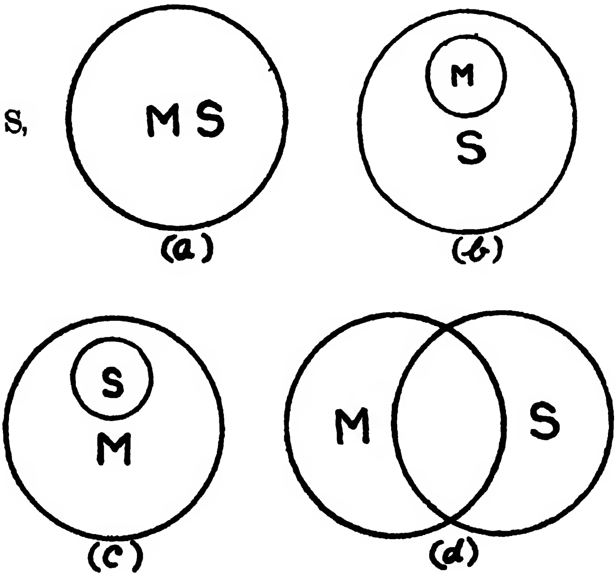
No P is M,
Some M is S,
∴ Some S is not P.

Here we have,

P e M,



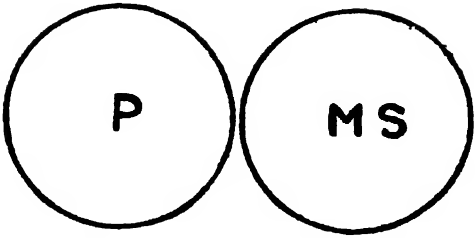

M i S,



Then,

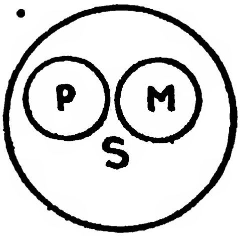
I. and (a)

give—

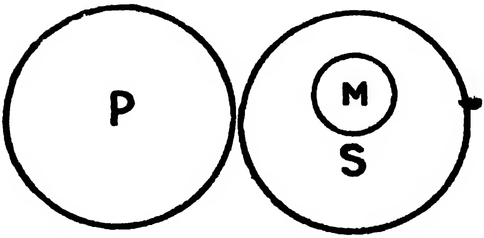



I. and (b)

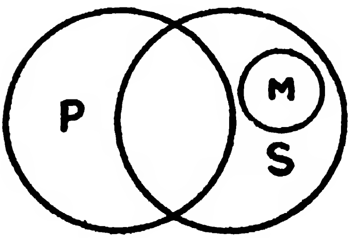
give—



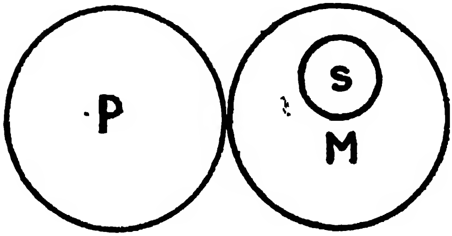

or



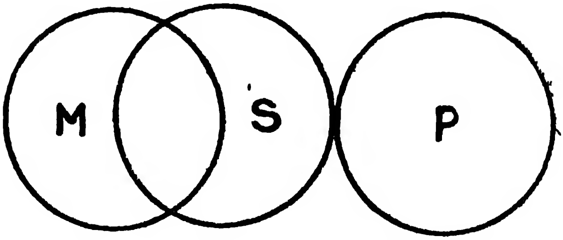

or



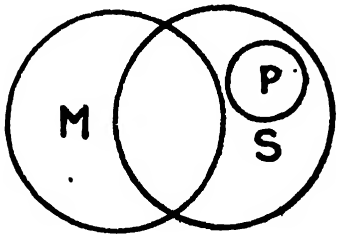
I. and (c) give—



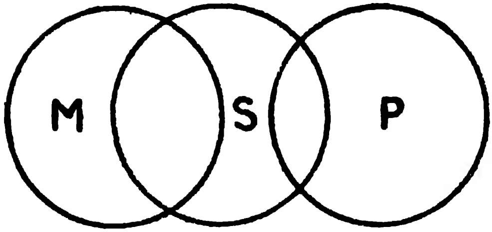

I. and (d) give—



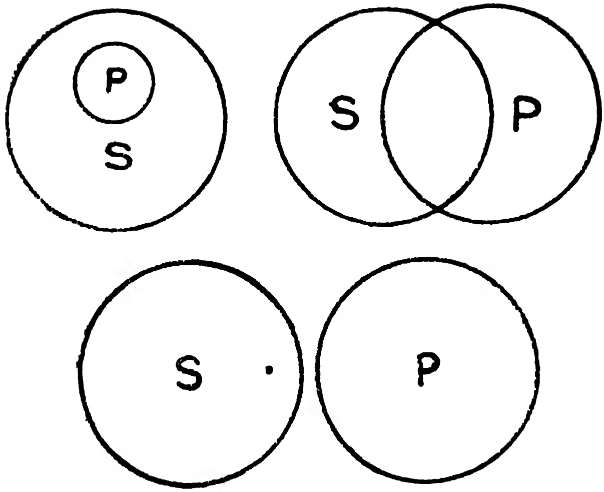

or




or




Neglecting M, we find that the above diagrams (which will be found to be the same as the diagrams of *Ferio,* though in a different order) reduce to three, *viz.,*—




from which, as in *Ferio,* we are justified in concluding that Some S is not P.

## HAPTER XIII

# MIXED SYLLOGISMS

### Mixed Hypothetical Syllogisms

We may now discuss the nature of *mixed syllogisms*. As already proposed, we shall discuss mixed syllogisms under three main heads, *viz.*, (1) *Mixed Hypothetical* or Hypothetico-categorial syllogisms, (2) *Mixed Disjunctive* syllogisms, and (3) the *Dilemma*. Jevons is of opinion that mixed hypothetical syllogisms should be simply called hypothetical syllogisms, since categorical and pure hypothetical syllogisms need not be distinguished, as they are governed by the same principles. Some call mixed hypothetical syllogisms hypothetico-categorical syllogisms. Fowler, Sigwart, Keynes, Welton and others distinguish between pure hypothetical and mixed hypothetical syllogisms. The former involve the same principles as categorical syllogisms, and we have seen that in them all the propositions are hypothetical, whereas mixed hypothetical syllogisms are governed by other principles. We therefore adopt this distinction between pure and mixed syllogisms so as to avoid confusion.

**Preliminary remarks.**

A *mixed hypothetical syllogism* is one in which the *major premise* is a *hypothetical* proposition and the *minor premise* and the *conclusion* are *categorical* propositions. To illustrate a mixed hypothetical syllogism we may express the major premise either by a hypothetical proposition or by a conditional one. The following is an example in which the major premise is an abstract hypothetical proposition: If P is true, Q is true (major premise); P is true (minor premise); ∴ Q is true (con-

**The nature of the mixed hypothetical syllogism explained and illustrated.**

clusion). In the following example the major premise is a conditional proposition: If any S is M, it is P (major premise); This S is M (minor premise); ∴ This S is P (conclusion). The following is a concrete example: If any man takes an overdose of strychnine he dies; Philip has taken an overdose of strychnine; ∴ He will die. In each of the above examples the minor premise affirms the antecedent, and the conclusion affirms the consequent. There is another form of mixed hypothetical syllogism in which the minor premise denies the consequent, and the conclusion denies the antecedent. Of this form the following are examples: If P is true, Q is true; Q is not true; therefore P is not true. If any S is M, it is P (major premise); This S is not P (minor premise); therefore This S is not M (conclusion). If any man is honest he is trusted; Ranjit is not trusted; therefore Ranjit is not honest. Though a distinction is made between hypothetical and conditional propositions, it is not important for our purpose, and we may regard both hypothetical and conditional propositions as hypothetical.

Canons of the mixed hypothetical syllogism and fallacies arising from their violation.

The two canons of mixed hypothetical syllogisms are: (1) The *assertion* of the *truth* of the *antecedent* of a hypothetical proposition *justifies* the *assertion* of the *truth* of the *consequent, but not conversely;* (2) the *denial* of the *consequent* necessitates the *denial* of the *antecedent,* but not conversely. In every hypothetical syllogism, the implication between one proposition and another is given in the major premise, and the relation between the antecedent and the consequent is one of ground and consequence. The first of the above rules may be proved thus:—A particular consequent may follow from different antecedents. Thus the occurrence of Q may follow the occurrence of A, B, or C. A man's death may be due to his taking poison or to his being attacked with some fatal illness. Now if the proposition is that 'If a man takes

poison, he dies,' we may, by affirming its antecedent, *viz.*, that this man has taken poison, affirm the consequent, *viz.*, that this man will die; but by affirming the consequent, *viz.*, that this man will die, we cannot affirm the antecedent, that this man has taken poison, because his death may be due to some other cause, say, to his being shot or to his being attacked with some fatal illness. The violation of the first rule, that is, an affirmation of the antecedent by affirming the consequent, gives rise to the fallacy of *affirming the consequent*. Similarly the second rule may be proved. Since the consequent of a hypothetical proposition may follow from different antecedents, we may, by denying the consequent, deny the antecedent, but we cannot, by denying the antecedent, deny the consequent. Thus if the proposition is, 'If any man gets into water, his body becomes wet', we can, by denying the consequent, *viz.*, that the body of the man is wet, deny the antecedent, *viz.*, that the man has got into water. But by denying the antecedent, *viz.*, that the man has got into water, we cannot deny the consequent, that his body is wet, for his body may get wet if he is exposed to rain or if somebody pours water upon his person. The violation of this rule gives rise to the fallacy of *denying the antecedent*.

**The two fundamental moods of the mixed hypothetical syllogism.**

The two main moods of mixed hypothetical syllogisms are: (1) *Modus ponens*, which is *constructive*, and (2) *Modus tollens*, which is *destructive*. A syllogism is in *modus ponens* when the *minor* premise *affirms* the *antecedent* of the *hypothetical major*, and the *conclusion affirms* its *consequent*. A syllogism is in *modus tollens* when the *minor* premise *denies* the *consequent* of the *hypothetical major*, and the *conclusion denies* its *antecedent*.

In every mixed hypothetical syllogism, the major premise states the principle, and the minor premise is subsumed under

it. Some logicians, such as Kant, Hamilton, and others regard the inference involved in mixed hypothetical syllogisms as immediate, but Keynes holds that the burden of proof is upon those who refuse to regard these syllogisms as syllogisms proper. Keynes also asserts that mixed hypothetical syllogisms in modus ponens are similar to the syllogisms in the first figure, because in them we pass from ground to consequence; while according to him syllogisms in modus tollens are similar to the moods in the second figure, since in them we pass from the denial of the consequence to the denial of the ground. Joseph however appears to be right when he asserts that what are usually called mixed hypothetical syllogisms are not syllogisms proper, but should rather be called hypothetical arguments, since in such reasoning there is no middle term, and the relation between the antecedent and the consequent of the hypothetical proposition in it is one of ground and consequence, not of subject and predicate.

**Are mixed hypothetical syllogisms syllogisms proper?**

German logicians provide four forms of modus ponens and four of modus tollens. Each of these two moods may have the following four forms:—(1) *Modus ponendo ponens,* (2) *Modus ponendo tollens,* (3) *Modus tollendo tollens,* (4) *Modus tollendo ponens.* These names are taken from the quality of the minor premise and the conclusion. In ponendo ponens both the minor premise and the conclusion are affirmative propositions. In ponendo tollens the minor premise is affirmative and the conclusion is negative. In tollendo tollens both the minor premise and the conclusion are negative. In tollendo ponens the minor premise is negative and the conclusion is affirmative. Let us now illustrate these forms of modus ponens and modus tollens respectively. The following are examples of the four forms of modus ponens: (1) *Ponendo ponens*—If A then B; A; therefore B.

**The four forms of the modus ponens and of the modus tollens.**

If a flower is sweet-scented it is pleasant; This flower is sweet-scented; ∴ This flower is pleasant. (2) *Ponendo tollens*—If A then not B; A; therefore not B. If a man is dishonest he is not trusted; This man is dishonest; ∴ This man is not trusted. (3) *Tollendo tollens*—If not A then not B; not A; therefore not B. If the finances of a country are not sound, its industries cannot prosper; the finances of Germany are not sound; therefore its industries cannot prosper. (4) *Tollendo ponens*—If not A then B; not A; therefore B. If the people of a country are not loyal to the government, its modification is inevitable; the people of China are not loyal to the government; therefore its modification is inevitable.

The following are the four forms of modus tollens: (1) *Ponendo ponens*—If not A then not B; B; therefore A. If a man is not intelligent he is not respected; Mr. Shastri is respected; therefore He is intelligent. (2) *Ponendo tollens*—If A then not B; B; ∴ not A. If South-Africa disobeys the decision of the U.N., it will not secure the co-operation of other states; South Africa has secured the co-operation of other states; therefore South Africa has not disobeyed the decision of the U.N. (3) *Tollendo tollens*—If A then B; not B; therefore not A. If England maintains friendly relations with Germany, she will lose the co-operation of France; England has not lost the co-operation of France; therefore England has not maintained friendly relations with Germany. (4) *Tollendo ponens*—If not A then B; not B; therefore A. If the market does not maintain a steady price, the traders are in difficulty; The traders are not in difficulty; ∴ The market has maintained a steady price.

We find that the names of the four forms of modus ponens are the same as those of modus tollens. There is indeed an identity between a form of modus ponens and a form of modus tollens having the same name. We may reduce either of them to the other. Let us take the modus

ponendo ponens of modus ponens, *viz.*, If A then B; A; therefore B. If we obvert the contrapositive of the major premise we get, If not B then not A. We may then have the following argument: If not B then not A; A; therefore B. This is modus ponendo ponens of modus tollens. Thus we find that the modus ponendo ponens of modus ponens is essentially the same as the modus ponendo ponens of modus tollens.

## Mixed Disjunctive Syllogisms.

Mixed disjunctive syllogisms defined, explained and illustrated.

*Mixed disjunctive* arguments, like mixed hypothetical arguments, though they are called syllogisms, cannot be regarded as syllogisms proper, since in them there is no middle term. Some logicians distinguish between disjunctive propositions, the form of which is S is not both P and Q, and alternative propositions, the form of which is S is either P or Q. We have not in our treatment laid much emphasis upon this distinction, and have described both forms by the same name, *viz.*, disjunctive propositions. Our discussion of the nature of these propositions in the previous book has already given hints as to the nature of disjunctive arguments. According to Keynes, "A *disjunctive* (or *alternative*) *syllogism* may be defined as a formal reasoning in which a *categorical premise* is combined with a *disjunctive* (alternative) *premise* so as to yield a *conclusion* which is *either categorical or* else *disjunctive* (alternative) *with fewer alternants than are contained in the disjunctive premise.*" In a mixed disjunctive syllogism or disjunctive-categorical syllogism as it is sometimes called, the *major* premise is a *disjunctive* proposition, the *minor* premise is a *categorical* one, the conclusion is either a categorical or a disjunctive proposition. When the *disjunctive* proposition has but *two* alternatives, the *conclusion* is *categorical, e.g.*, S is either P or Q, S is not P, therefore S is Q; or, Either S is P or

S is Q, S is not P, therefore S is Q. When the *alternatives* are *more than two,* the *conclusion* is a *disjunctive proposition, e.g.,* S is either P or Q or M; S is not P; therefore S is either Q or M. Such a form of disjunctive proposition is said to be in the modus tollendo ponens, because the minor premise denies one of the alternatives and the conclusion affirms the other alternative or alternatives. We have in a previous chapter pointed out that where the alternatives of a disjunctive proposition are exhaustive but not exclusive, the modus tollendo ponens is legitimate. If we take it for granted that the alternatives 'intelligent' and 'industrious' are the only alternatives by which we can account for the striking success of a man, say X, we may argue in this way: X is either intelligent or industrious; He is not intelligent; therefore He is industrious. In this case the alternatives are not exclusive, and therefore by affirming one alternative we cannot deny the other, because X may be both intelligent and industrious at the same time.

**Disjunctive syllogisms in the modus tollendo ponens explained.**

A disjunctive syllogism in the modus tollendo ponens may have four forms—(1) X is either P or Q; X is not P; therefore X is Q; (2) X is either P or not Q; X is not P; ∴ X is not Q; (3) X is either not P or not Q; X is P; ∴ X is not Q; (4) X is either not P or Q; X is P; ∴ X is Q. Each of the above arguments can be reduced to a mixed hypothetical syllogism, and we then find that the arguments 1, 2, 3 and 4 are respectively in modus tollendo ponens, modus tollendo tollens, modus ponendo tollens and modus ponendo ponens. Of the above four forms, the first may be thus reduced to a hypothetical syllogism: If X is not P, it is Q (major); X is not P (minor); ∴ X is Q. We need not reduce the other three forms of disjunctive syllogism to hypothetical ones. Since disjunctive syllogisms can be reduced to hypothetical ones, which again can be reduced to categorical syllogisms, there is an essential unity between

them all. The canon of the above form of the disjunctive syllogism is, To deny one (or more) of a number of alternatives of a disjunctive proposition is to affirm the remaining alternative or alternatives. Joseph rightly remarks that the use of a disjunctive argument lies more in what it can establish than in what it can overthrow. From the above discussion it is clear that disjunctive syllogisms in modus tollendo ponens rest upon the law of excluded middle.

**Disjunctive syllogisms in modus ponendo tollens explained.**

We have also shown in the previous chapter that when the alternatives of a disjunctive proposition are exclusive but not exhaustive, we can, by affirming one of the alternatives, deny the other, but by denying one of the alternatives we cannot affirm the other. Thus the proposition, This flower is either green or red, is a disjunction of which the alternatives are exclusive but not exhaustive. With such a disjunctive major premise we may have the following argument: This flower is either green or red; This flower is green; ∴ This flower is not red. Here by denying one of the alternatives we cannot affirm the other, because the flower may be yellow. Such disjunctive syllogisms as the one given above are in modus ponendo tollens. They also may have four forms, *viz.*, (1) S is either P or Q; S is P; ∴ S is not Q; (2) S is either P or not Q; S is P; ∴ S is Q; (3) S is either not P or not Q; S is not P; ∴ S is Q; (4) S is either not P or Q; S is not P; ∴ S is not Q. When the disjunctive arguments are reduced to hypothetical forms, the above forms 1, 2, 3, 4 are found to be in modus ponendo tollens, modus ponendo ponens, modus tollendo ponens and modus tollendo tollens, respectively. The canon of the disjunctive syllogism in modus ponendo tollens is, To affirm one member (or more) of any alternative is to deny the other member or members. Disjunctive syllogisms in modus ponendo tollens rest upon the law of contradiction. We may observe that when the alternatives of a disjunctive propo-

sition are both exclusive and exhaustive, as in the case 'This flower is either white or not white', we may, by denying one of the alternatives, affirm the other, and by affirming one of the alternatives deny the other. With such a disjunctive proposition as the major premise, we may have disjunctive syllogisms both in the modus tollendo ponens and in the modus ponendo tollens.

### Dilemma

**Dilemma defined and its nature stated.**

We have already discussed two forms of what is called the mixed syllogism, and may now consider its remaining form, *viz.*, the *Dilemma,* which is more complex than either the hypothetical or the disjunctive forms of the mixed syllogism. If it be true that neither hypothetical nor disjunctive arguments are syllogisms proper, it is also true that dilemmas are not properly speaking syllogisms. though they are so called. A *dilemma* may be defined as a formal argument containing a *premise* in which *two hypothetical propositions* are *conjunctively affirmed,* and a *second premise* which is a *disjunctive proposition* in which the *antecedents* of these *hypotheticals* are *alternatively affirmed, or* their *consequents alternatively denied.* Thus a dilemma has as its major premise. two hypothetical propositions with two distinct antecedents and a common consequent, or two distinct consequents and a common antecedent, or two distinct antecedents and two distinct consequents. The minor premise may either affirm the antecedents or denv the consequents alternatively, and the conclusion will affirm the consequent or consequents as the case may be, or deny the antecedent or antecedents as the case may be. (When the *conclusion* affirms only *one* consequent or denies *one* antecedent, it is a categorical proposition, and the dilemma is then *simple.*) When the *conclusion* affirms *two* alternative consequents or denies two alternative antecedents, the dilemma is then *complex,* and the conclusion is a disjunctive proposition. Further, (when

the *minor* premise *affirms* the antecedents and the *conclusion affirms* the consequent or consequents, the dilemma is said to be *constructive,* and it may be said to be in modus ponens. But when the *minor* premise *denies* the consequents and the *conclusion denies* the antecedent or antecedents, the dilemma is regarded as *destructive* and may be said to be in modus tollens.

We thus find that the dilemma combines into one argument both the hypothetical and the disjunctive modes of reasoning. The aim of a dilemma is to prove something against an opponent. and it is therefore unpalatable and disagreeable to him. It is therefore defined by Joseph as "a hypothetical argument offering alternatives, and proving something against an opponent in either case." We have a dilemma when the major premise offers two antecedents or two consequents and the minor premise alternatively affirms or denies them. When the major premise contains three distinct antecedents or three distinct consequents, and the minor premise affirms three alternative antecedents or denies three alternative consequents respectively, we have what is known as a Trilemma. When the alternatives affirmed or denied in the minor premise are four in number, we have a Tetralemma, and when they are more than four in number we have what is known as the Polylemma. Since the Trilemma, Tetralemma, and Polylemma do not involve any new principle not operative in the dilemma we need not treat of them separately.

We have already seen that a dilemma may be either simple or complex, and it may also be either constructive or destructive. So in the main there are four forms of the dilemma:—I. Simple constructive.—*A simple constructive dilemma* is one in which the *major* premise, which contains *two hypotheticals,* provides *two* distinct *antecedents* and a *common consequent*; the *minor* premise *alternative-*

Simple constructive dilemma illustrated.

*ly affirms* the *antecedents*; and the *conclusion affirms* the *consequent. e.g.,* If A is B, X is Y, and if C is D, X is Y (major premise); Either A is B or C is D (minor premise); therefore X is Y (conclusion). Joseph gives an interesting concrete example of this form: Troops having an impassable river behind them and a deadly enemy in front may be faced with the following dilemma: "If they stand their ground they die—by the sword of the enemy; if they retreat they die—by the flood; but they must either stand or retreat; therefore they must die." Thus we find that whatever alternative of a dilemma is accepted, the result is unpleasant; hence the saying 'to be on the horns of a dilemma.' An opponent who is faced by a dilemma is between the devil and the deep sea.

**Complex constructive dilemma illustrated.**

II. Complex constructive.—In a *complex constructive dilemma* the *major* premise contains *two* distinct *antecedents* and *two* distinct *consequents*; the minor premise *alternatively affirms* the *antecedents*; and the *conclusion alternatively affirms* the *consequents*; *e.g.,* If A is B, X is Y; and if C is D, M is N; Either A is B or C is D; therefore Either X is Y or M is N. The following is a concrete example: "If there is a censorship of the press, abuses which should be exposed will be hushed up; and if there is no censorship, truth will be sacrificed to sensation; but there must either be a censorship or not; therefore either abuses which should be exposed must be hushed up, or truth be sacrificed to sensation" (Joseph).

**The simple destructive dilemma illustrated.**

III. Simple destructive.—A *simple destructive* dilemma is one in which the *major* premise contains a *common antecedent* and *two* distinct *consequents*; the *minor* premise *alternatively denies* the *consequents*; and the *conclusion denies* the *antecedent*; *e.g.,* If A is B, X is Y, and if A is B, M is N; Either X is not Y or M is not N; therefore

A is not B. The following is a concrete example: "If Homer speaks truth about things divine, the heroes were sons of gods, and did many wicked deeds; but either they were not sons of gods, or they did not do wicked deeds; therefore Homer does not speak truth about things divine" (Joseph).

IV. Complex Destructive.—A *complex destructive* dilemma is one in which the *major* premise contains *two* distinct *antecedents* and *two* distinct *consequents*; the *minor* premise *alternatively denies* the *consequents*; and the *conclusion alternatively denies* the *antecedents*; *e.g.* If A is B, X is Y, and if C is D, M is N; Either X is not Y or M is not N; therefore Either A is not B or C is not D. The following is a concrete example:—"If we give our colonies self-government, we shall make them powerful; and if we attempt to control their use of it, we shall make them hostile; But either we ought not to make them powerful, or we ought not to make them hostile; therefore Either we ought not to give them self-government or we ought not to attempt to control their use of it." (Joseph).

**The complex destructive dilemma illustrated.**

Mansel, and some other logicians following him, hold that destructive dilemmas are always complex and cannot be simple. But our previous discussion shows that destructive dilemmas can legitimately be simple.

Jevons maintains that a dilemmatic argument is more often fallacious than not, because the alternatives are not usually exhaustive. But this view does not appear to be acceptable. A wrong notion persists that the reasoning involved in the dilemma is not sound. But the formal validity of a dilemma cannot be questioned if its structure is sound, that is, if its major premise, minor premise and conclusion are what they ought to be according to the definition of the dilemma. A dilemmatic argument may however be materially false, if in the major premise the consequent

**A dilemmatic argument need not be always false.**

does not follow from the antecedent, or if the alternatives are not exhaustive. A dilemma which is materially false can be rebutted or refuted by a counter-dilemma. Thus the following dilemma—If A is B, X is Y, and if C is D, M is N—can be rebutted by the dilemma, If A is B, M is not N, and if C is D, X is not Y. Only a complex constructive dilemma can be rebutted. The alternatives of a dilemma can be exhaustive only when they are contradictories, but often they are not so. When they are not exhaustive, a man may escape between the horns of a dilemma, that is, between the alternatives. Thus an attempt is made to refute the famous dilemma by which Zeno disproved the existence of motion, which is : If a body moves, it must either move in the place where it is, or in the place where it is not ; But it can neither move in the place where it is, nor in the place where it is not ; therefore It cannot move. In reply to this dilemma, it is pointed out that a body need not move either where it is or where it is not, but it may move between these two places. But this attempt to refute the dilemma is unsuccessful, because if the body moves at all, it must move either where it is or where it is not.

**Three ways of refuting a dilemma, viz. by rebutting it, by escaping between its horns, or by taking the dilemma by its horns.**

A third way of refuting a dilemma is to take it by the horns, that is, to show that the consequent or consequents do not follow from the antecedents.

In such a case the major premise can be replaced by another premise which refutes the former, and in which the consequent or consequents follow from the antecedents. If anybody argues thus: 'If you take milk, you will suffer from indigestion, and if you take bread, you will suffer from indigestion ; but You must either take milk or bread ; therefore You must suffer from indigestion,' we may refute him by the counter-dilemma, 'If a person takes milk he will be vigorous, and if he takes bread he will be vigorous ; He

must either take milk or bread; therefore He must be vigorous.' This dilemma may also be refuted by pointing out that the person may take fruit instead of taking either milk or bread, and thus there is an escape between the horns of the dilemma.

In the preceding paragraphs we have shown that in order to rebut a dilemma we combine the antecedents of each of the two hypothetical propositions which form the major premise, with the contrary or contradictory of the consequent of the other. We have also observed that only a complex constructive dilemma can be rebutted. The following is a classical example of how a dilemma is rebutted:—

Some classical examples of the dilemma.

An Athenian mother wanted her son not to enter public life, and advanced the following dilemma to dissuade him: "If you act justly, men will hate you, and if you act unjustly, the gods will hate you; but you must act either justly or unjustly; therefore, public life must lead to your being hated". The son rebutted the above dilemma by the following: "If I act justly the gods will love me, and if I act unjustly men will love me; I must act either justly or unjustly; therefore, entering public life will make me beloved." It may be pointed out that the two conclusions are not really incompatible, because a public man is always both hated and loved.

We may conclude this topic with two other classical examples of the dilemma. *Litigiosus* is a famous dilemma Protagoras consented to give lessons to Euathlus in rhetoric, and it was agreed that one-half the fee should be paid at once and the other half when Euathlus would win his first case. When Protagoras found that Euathlus engaged in no suit, he sued him and advanced the following dilemma: "If you lose this suit you must pay me by order of the court, and if you gain it you must pay me by our contract." To which Euathlus retorted: "If I lose this suit I am free from

payment by our contract, and if I gain it, I am exonerated by the judgment of the court." The best solution of this difficulty is that since Euathlus had won no case up to that time, the judges would decide in his favour. After this Protagoras, when he saw Euathlus had won a case, might sue him again and might reasonably expect that the judges would decide in his favour. Another famous dilemma is known as *Crocodilus*. A crocodile seized a child and told its mother that he would give it back if the mother could say correctly whether he would give it back or not. Fearing that if she said he was going to give it back he would prove her wrong by devouring it, she replied that he would not give it back, and argued: "Now you must give it back; for if my answer is true you must give it back in accordance with your promise, and if the answer is false you must give it back to prevent its being true." The crocodile replied: "I will not give it back, for if I did, your answer would be false and I should break our agreement; and even if your answer were correct I could not give it back, as that would make your answer false." There seems to be no way out of this dilemma, but the answer would have been more fortunate if the mother had said that he would give the child back, for in that case its restoration would both have made her answer true and have fulfilled the agreement.

# CHAPTER XIV

## THE ENTHYMEME, SORITES AND EPICHEIREMA

### The Enthymeme

The arguments to be considered in this chapter do not involve any principles other than those considered in connection with the syllogism. We should regard an Enthymeme as a particular way of argument rather than as a new form of argument. The *Enthymeme,* according to Aristotle, is a rhetorical syllogism, as distinguished from the apodeictic demonstrative and theoretical syllogism. In an enthymematic argument one of the premises or the conclusion of a syllogism is understood and not expressed. It is therefore a syllogism incompletely stated. So Welton defines an Enthymeme as "A *syllogism abridged* in expression by the *omission* of one of the *constituent propositions."* In ordinary discourse syllogistic arguments are not usually expressed in full, and the arguments of everyday life are, to a large extent, enthymematic. Outside the treatises on logic we hardly meet with syllogisms in which all the constituent propositions are expressed. People often have recourse to enthymemes with a view to making a fallacious argument appear true. It is therefore often a very useful means to cover up fallacies in reasoning. Sometimes however things are so well understood between contending parties as not to require any explicit statement. But such shortcuts often serve to confuse and lead to fallacies.

**The Enthymeme defined.**

**Enthymematic arguments common in ordinary discourse.**

When the *major premise* of an *enthymeme* is *suppressed,*

it is said to be an enthymeme of the *first order*; when the *minor* premise is *suppressed* it is said to be of the *second order*; and when the *conclusion* is *omitted*, it is said to be of the *third order*. 'Every soul is a spiritual substance, ∴ No soul is tangible,' is an enthymeme of the first order, because here the major premise, 'No spiritual substances are tangible,' has been suppressed. 'No spiritual substances are tangible. and ∴ No soul is tangible,' is an enthymeme of the second order, because here the minor premise, 'Every soul is a spiritual substance,' has been suppressed. 'No spiritual substances are tangible, and Every soul is a spiritual substance,' is an enthymeme of the third order, because here the conclusion, 'No soul is tangible,' has been suppressed. Similarly, 'John is ambitious, therefore he is unhappy,' is an enthymeme of the first order; 'All ambitious men are unhappy, and therefore John is unhappy,' is an enthymeme of the second order; while 'All ambitious men are unhappy, and John is ambitous,' is an enthymeme of the third order. We need not multiply instances to explain the nature of enthymemes of different orders.

**The three orders of enthymemes explained and illustrated.**

Enthymemes may consist of hypothetical, as well as, of categorical propositions. Joseph observes that "A syllogism, whether expressed in full or as an enthymeme, is a single act of inference; it may be analysed into premises and conclusion, but not into parts which are themselves acts of inference."

## Episyllogistic and Prosyllogistic Trains of Reasoning

A *polysyllogism* or train of syllogisms is a *combination* of a number of *syllogisms,* in which the *conclusion* of *one* syllogism serves as the *premise* of *another.* Such a train of syllogisms may consist either of categorical propositions or of hypothetical ones. Thus according to Keynes, "A chain of syllogisms, that is, a series of syllogisms so linked together that the conclusion of one becomes a premise of

**The nature of a polysyllogism or train of syllogisms.**

another, is called a polysyllogism". In a polysyllogism, that syllogism the conclusion of which is a premise of another syllogism is called, in relation to the latter, a *prosyllogism*; while the syllogism which uses the conclusion of another syllogism as one of its premises is called, in relation to that other syllogism, an *episyllogism*. A syllogism which in relation to one syllogism is a prosyllogism may be an episyllogism in relation to another. We may illustrate the matter by an example: A is C, B is A, ∴ B is C; B is C, D is B, ∴ D is C; D is C, E is D, ∴ E is C. In this train, B is C, D is B, ∴ D is C is a prosyllogism in relation to the syllogism D is C, E is D, ∴ E is C, because the latter syllogism uses the conclusion of the former as one of its premises; but it is an episyllogism in relation to the syllogism A is C, B is A, ∴ B is C, because one of its premises is the conclusion of the latter syllogism. Thus we find that a prosyllogism proves a premise of an episyllogism. The following is a concrete example: All material bodies are subject to decay, Animal bodies are material, ∴ Animal bodies are subject to decay; Human bodies are animal bodies, ∴ Human bodies are subject to decay; The body of John is a human body, ∴ The body of John is subject to decay.

**A prosyllogistic train of reasoning illustrated.**

The trains of reasoning given above are called synthetic, episyllogistic or progressive. In a progressive train of syllogisms thought moves from prosyllogism to episyllogism. A train of reasoning is said to be regressive, analytic or prosyllogistic when thought moves from episyllogism to prosyllogism. The following is an example of a prosyllogistic train: All men are mortal; Socrates is a man; ∴ Socrates is mortal; all animals are mortal; all men are animals; ∴ all men are mortal; all living beings are mortal; all animals are living beings; ∴ all animals are mortal. In this train the syllogism last stated proves a premise of the syllogism stated

**An episyllogistic train of reasoning illustrated.**

before it, and this syllogism again proves a premise of the syllogism stated first. In actual occurrence of thought, both progressive and regressive trains of reasoning are met with. Thought advances from prosyllogism to episyllogism when the most general principle is stated first, and it moves from episyllogism to prosyllogism when the most general principle is stated at the end.

### The Sorites

*Sorites* is a *progressive,* episyllogistic or synthetic *train of reasoning* composed of a *number of enthymemes.* Keynes defines it as "a polysyllogism in which all the conclusions are omitted except the final one, the premises being given in such an order that any two successive propositions contain a common term." In a sorites the first syllogism of the series or chain is an enthymeme of the third order, while the last is an enthymeme of either the first or the second order. There are *two forms* of sorites, *viz.*, the Aristotelian and the Goclenian, which may be illustrated by the following symbolic examples: Aristotelian sorites—A is B, B is C, C is D, D is E, ∴ A is E: Goclenian sorites—D is E, C is D, B is C, A is B, ∴ A is E. The order of the Goclenian sorites is the reverse of the order of the Aristotelian. In the Aristotelian sorites the term which is common to any two successive premises occurs as predicate in the premise stated first and as subject in that which follows; while in the Goclenian sorites the term which is common to any two successive premises occurs first as subject and then as predicate. Further, in the Aristotelian sorites the premise which is stated first in the series contains the subject of the conclusion, and the premise stated last the predicate of the conclusion; while in the Goclenian sorites the first premise of the chain contains the predicate of the conclusion, and the

Sorites defined.

Sorites illustrated by symbols.

The Aristotelian and Goclenian sorites compared.

last the subject. It will be found immediately, when we fully draw out the Aristotelian and Goclenian sorites, that in the former the premise stated first and all the suppressed premises are minor premises, while in the latter the premise stated first and all the suppressed premises are major premises. We may now develop the two forms of sorites illustrated above, to make clear what we have stated in the preceding lines. Aristotelian sorites—B is C, A is B, ∴ A is C; C is D, A is C, ∴ A is D; D is E, A is D, ∴ A is E. Goclenian sorites:—D is E, C is D, ∴ C is E; C is E, B is C, ∴ B is E; B is E, A is B, ∴ A is E.

The fully developed forms of the sorites illustrated above.

We may now cite a concrete instance to illustrate the sorites. The following example is given by Aristotle: Action is that in which happiness lies; what contains happiness is the end and aim; the end and aim is what is highest; therefore action is what is highest. The above example gives us the Aristotelian form of sorites, and we may obtain the Goclenian form by reversing the order of the premises as follows: The end and aim is what is highest; what contains happiness is the end and aim; action is that in which happiness lies; therefore action is what is highest. The following statement of St. Paul in the *Romans* provides an example of the sorites: "For whom he did foreknow, he also did predestinate to be conformed to the image of his son.... ....Moreover whom he did predestinate, them he also called; and whom he called, them he also justified; and whom he justified, them he also glorified." The above is an Aristotelian sorites of which the final conclusion is not stated, and if the premises are reversed as before, we may have the form of the Goclenian sorites.

Some concrete examples.

There may be sorites consisting of hypothetical propositons. Welton gives the following two examples:—"If any

The sorites consisting of hypothetical propositions illustrated.

man is avaricious he is intent on increasing his wealth; if he is so intent, he is discontented; if he is discontented, he is unhappy; therefore if any man is avaricious, that man is unhappy." In this case all the constituent syllogisms are pure hypothetical ones, while in the following example the last syllogism is a mixed hypothetical syllogism: "If the soul thinks, it is active; if it is active, it has strength; if it has strength, it is a substance; now the soul thinks; therefore the soul is a substance." We may here remark that both Aristotelian and Goclenian sorites are progressive, and Goclenian sorites should not be regarded as a regressive train of reasoning, as some logicians suppose it to be. (According to Hamilton the Aristotelian sorites is an argument in comprehension, while the Goclenian sorites is an argument in extension.)

Rules of the Aristotelian sorites stated.

We may now state the *rules* of *sorites.* The two *rules* of the *Aristotelian* sorites are: (1) only *one* premise can be *negative,* and if one is negative, it must be the *last;* (2) only *one* premise can be *particular,* and if one is particular, it must be the *first.* The above rules may be proved thus:—If two premises of a sorites are negative, then we shall come across a syllogism in the series consisting of two negative premises, which we have found is not allowed by a general rule of the syllogism. Again if one of the premises of a sorites is negative, the conclusion must be negative, and the predicate of the conclusion will therefore be distributed. So the term which is the predicate of the conclusion has to be distributed in the premise in which it occurs. We have found that in the Aristotelian sorites the predicate of the conclusion is the predicate of the premise stated last, and it can be distributed if that premise is negative. So it is proved that if one premise is negative, it must be the last, in the Aristotelian sorites. Further, if more than one premise is particular in a sorites, we shall come across a syllogism in the series with two

particular premises, which is not allowed by a general rule of the syllogism. Further, if the last premise of an Aristotelian sorites be particular instead of the first, then there will be the fallacy of undistributed middle, because the last premise is the major premise, of which the middle term is the subject, and it can be distributed only when it is universal. The other premise, which is suppressed, is the minor premise, and has the middle term as its predicate, and this premise being affirmative (since the last premise alone in the Aristotelian sorites can be negative) cannot distribute the middle term.

The rules of the Goclenian sorites stated and proved.

The following are the two *rules* of the *Goclenian* sorites:—(1) only *one* premise can be *negative*, and if one is negative, it must be the *first*; (2) only *one* premise can be *particular*, and if one is particular, it must be *last*. If more than one premise in this form of sorites is negative, we have a syllogism with two negative premises, as in the case of the Aristotelian sorites, which is not allowed by a general rule of the syllogism. If one of the premises is negative, the conclusion will be negative, and the predicate of the conclusion will be distributed. The term which is the predicate of the conclusion has therefore to be distributed in the premise in which it occurs. We have found that in the Goclenian sorites the first premise in the series has for its predicate the term which is the predicate of the conclusion. So the first premise must be negative, if any premise in the Goclenian sorites is to be negative, in order to distribute the required term. If more than one premise is particular in this form of sorites, we shall come across a syllogism in the syllogistic train consisting of two particular premises which cannot be allowed. If one of the premises is particular in the Goclenian sorites, it must be the last, which, we have found, cannot be negative and is the minor premise of the last syllogism. The middle term in this premise, being the predicate of an affirmative proposition, cannot be distributed. Therefore it must be distributed in

the suppressed major premise. The middle term is the subject of this premise, and can be distributed only if it is universal. Therefore if any premise in the Goclenian sorites be particular, it must be the last, otherwise there will be the fallacy of undistributed middle.

Logicians like Mill and Keynes hold that there may be sorites in figures two and three. We need not give examples of such sorites. The rules of sorites do not apply to them.

**The Epicheirema**

The Epicheirema defined and its four forms explained and illustrated.

"An *Epicheirema* is a *regressive* chain of reasoning abridged by the *omission* of *one* of the *premises* of *each prosyllogism*" (Welton). It is therefore a polysyllogism with only one or more prosyllogisms brefly indicated. It being a regressive, analytic or prosyllogistic train of reasoning, the movement of thought in the epicheirema is from episyllogism to prosyllogism. The epicheirema may be either *single* or *double,* and it may be either *simple* or *complex.* So it may have four forms, *viz.,* (1) simple single, (2) simple double, (3) complex single, (4) complex double. We may illustrate these forms by symbolic examples:—

(1) *Simple single*:—Every M is P because it is X: Every S is M; Therefore S is P. This is single because only the major premise of the argument is proved by a prosyllogism, one premise of which is suppressed. If we fully express the prosyllogism which proves the major premise of the episyllogism, we find it to be—Every X is P (suppressed major premise); Every M is X; therefore Every M is P. Thus we find that the prosyllogism which, in this case, proves one of the premises of the episyllogism, is an enthymeme of the first order. Let us take another example:—M is P because X is; S is M; ∴ S is P. In this case, if the prosyllogism, which is an enthymeme, is fully developed, we have the argument: Every X is P (major premise); Every M is X (suppressed minor

premise); ∴ Every M is P. In this case the prosyllogism which proves one of the premises of the episyllogism is an enthymeme of the second order.

(2) *Simple double*:—The following are two symbolic examples of this form of the epicheirema: (*a*) Every M is P, because it is X; Every S is M, because it is Y; therefore Every S is P. (*b*) Every M is P, because every X is; Every S is M, because every Y is; therefore Every S is P. In the first example, the prosyllogisms which prove the two premises of the episyllogism are enthymemes of the first order, while in the second example they are enthymemes of the second order. An epicheirema is double when both the premises of the episyllogism are proved by enthymemes.

(3) *Complex single*:—An epicheirema is complex when its premise is proved by an enthymeme which again is proved by another enthymeme. If only one of the premises is so proved, we have the single complex epicheirema, and if both the premises are so proved we have the double complex epicheirema. The following is an example of a single complex epicheirema:—Every M is P, because it is X, and every X is Y; Every S is M; therefore Every S is P.

(4) *Complex double*:—The following is an example of the double complex epicheirema: Every M is P, because it is X, and every X is Y; Every S is M because it is N, and every N is O; therefore Every S is P.

We may give a concrete example of the simple double epicheirema: All ambitious men are unhappy because they can never satisfy their desires, and all statesmen are ambitious because they always want to be more powerful; therefore all statesmen are unhappy. We need not cite many more concrete examples to illustrate the epicheirema. We may conclude with one further concrete example of it given by Joseph: "Those who have no occupation have nothing to interest themselves in, and therefore are unhappy; for men with

Some concrete examples.

nothing in which to interest themselves are always unhappy, since happiness depends on the success with which we advance the objects in which we are interested; and so wealth is no guarantee of happiness." Here the central syllogism is: All who have nothing in which to interest themselves are unhappy; Those who have no occupation have nothing in which to interest themselves; therefore those who have no occupation are unhappy.

# CHAPTER XV

## FUNCTION, VALIDITY AND RANGE OF THE SYLLOGISM

In this chapter we shall consider whether syllogistic reasoning is useful, whether it involves the fallacy of petitio principii, and what its scope is. We shall be able to determine the problem of the value and validity of the syllogism if we clearly understand its function. For a long time it has been argued that the syllogism is either useless or fallacious. Mill puts the matter thus: "If all the facts of the major premise of any syllogism have been examined, the syllogism is needless; and if some of them have not been examined, it is a petitio principii. But either all have been examined, or some have not. Therefore the syllogism is either useless or fallacious."

Problems to be discussed in this chapter.

We must here remember that every useful and valid inference must satisfy two conditions, *viz.*, (1) it must contribute to the advancement of knowledge, that is, the conclusion must be different from the premises; and yet (2) it must follow necessarily from the premises. Thus the paradox of inference is that the conclusion arrived at by means of it must be within the premises and yet outside them, that is to say, it must go beyond the premises even though necessarily implied by them. Does the syllogistic inference satisfy these two conditions? If it does not, it is certainly useless and invalid. In every syllogism there is a universal proposition, and we have seen that the mood Barbara is the ideal type of syllogism. It is held by the opponents of syllogism that, in it the conclusion does not go beyond the premises, and therefore it is of little value. Let us now see what really is the

The syllogism cannot be regarded as useless.

function of syllogism, by taking an example of it and asking whether it is useful. 'All material bodies gravitate, This stone is a material body, ∴ It gravitates', is a perfect syllogism in the mood Barbara. The opponents of the syllogism hold that, if we know that all material bodies gravitate, we also know that this stone gravitates; thus the conclusion does not furnish any new information or go beyond the premises, and is therefore of little use and not, properly speaking, an inference at all. Is this contention true? It appears that the opponents of the syllogism are wrong. The major premise of a syllogism need not be an enumerative universal, that is, it need not be established by an observation of all the instances comprehended under it. In the illustration given above, the major premise, 'All material bodies gravitate,' is a true abstract universal, because it has been established on the basis of some universal principle after observing only a few instances of material bodies gravitating. Thus, though we are aware of the major premise of the syllogism, the conclusion drawn from it may really provide some new information to the enquiring mind, and in this sense it goes beyond the premises, though it necessarily follows from them.

Inference is a movement of thought, and its claim to value depends upon subjective, not upon objective, conditions. We have seen that even in immediate inference there is a movement of thought, and it is therefore inference proper. Mere verbal novelty is not sufficient for inference. If we pass from the proposition that 'George VI is the king of England' to the proposition that 'George VI is England's king', there is no inference at all, as there is no movement of thought from the known to the unknown. It is merely a verbal change. But in syllogistic reasoning this is not so. The novelty that is required of syllogistic reasoning is subjective novelty, as Keynes puts it, not objective novelty or mere verbal novelty. Let us take the syllogism, 'All ruminants are herbivorous, Camels are ruminants, ∴ Camels are herbivorous.' Here we

may be aware of the major premise without being aware of the conclusion, though the former implies the latter; and this is certainly a case of inference proper. Therefore the contention that the syllogism is useless cannot be established. We may also point out that in a syllogism we do not draw a conclusion from the major premise alone; the minor premise also is necessary. This is recognised both by the dictum of Aristotle and by the dictum nota notæ, as we have pointed out in our discussion of the syllogism. If this is so, that is, if the minor premise in an indispensable necessity of the syllogistic argument, there is an additional reason to reject the contention that it is useless, since the conclusion is not drawn from the major premise alone.

Mill's view of the syllogism examined.

Let us now examine Mill's contention that all arguments are from particulars to particulars. He maintains that syllogistic arguments are not, properly speaking, cases of inference. They are simply interpretative. Our arguments are always free from the necessity of having recourse to the universal. We argue that John is mortal, James is mortal, Philip is mortal, and so on, therefore the Duke of Wellington is mortal. We need not infer the mortality of the Duke of Wellington from the premise 'All men are mortal.' The universal proposition of a syllogism is nothing but a memorandum of observed facts. We may, no doubt, argue in this way that John is mortal, James is mortal, Philip is mortal, and so on, therefore All men are mortal; and the Duke of Wellington is a man, and therefore he is mortal. But such a process of thought is not necessary to arrive at this conclusion, because we can pass directly from the observed facts to the ultimate conclusion without having recourse to the universal proposition 'All men are mortal.' Thus Mill writes, "I cannot perceive why it should be impossible to journey from one place to another unless we march up a hill and then march down again." "Not only may we

reason from particulars to particulars without passing through generals, but we perpetually do so reason." "The child who, having burnt his fingers, avoids to thrust them again into the fire, has reasoned or inferred, though he has never thought of the general maxim, Fire burns." But Bradley rightly points out that no inference is possible without the help of some universal element. Even in induction, which Mill so vehemently advocates, it is impossible to establish a valid conclusion without perceiving the universal connection between the subject of inference and the inferred property. Even when Mill's village matron finds out what the illness of a neighbour's child is from the illness of her own child which she observed in the past, she can correctly do so only if she sees the universal connection between the illness and its symptoms. Only in an analogical argument do we pass from the particular to the particular without the help of any universal element. But such arguments, as we shall later on find, are more often fallacious than not.

Since it is impossible to do away with the universal element in reasoning, it is also impossible to dispense with the need of the syllogism. If the syllogistic argument is nothing but the interpretation of the major premise in the conclusion, then even geometrical inference must be regarded as merely interpretative and not as reasoning proper. But in the deductive reasonings of geometry, though conclusions necessarily follow from self-evident axioms and postulates, it cannot be contended that they are apprehended as soon as the axioms and postulates are known. Even if we know the latter, we are not aware of many of the conclusions that are established from them. We have already shown that there is a real advancement of knowledge in syllogistic arguments. By means of syllogisms the laws of Kepler are extended to newly discovered planets and satellites, which had not been observed when the laws were established. Even Mill admits that the argument, 'All men are mortal, the Duke of Wellington is a man, there-

fore the Duke of Wellington is mortal,' is an argument the conclusion of which is not apprehended as soon as the major premise is given, since when he was writing his *Logic* the Duke of Wellington was still alive. Even Bradley admits that three-fourths of our reasonings are syllogistic. Thus we find that the sylogism is not useless, for the conclusion is not obviously apprehended as soon as we apprehend the major premise, and the minor premise is indispensable. Only when the major premise of a syllogism is an enumerative universal, that is, is established by observing all the instances comprehended under it, can it be regarded as not very useful. We shall find, when we discuss the validity of induction, that the general proposition established by induction is only hypothetical, and can be verified and proved only when tested by syllogistic reasoning.

The conclusitn to which our arguments lead

The above considerations will now enable us to establish that the syllogistic argument is not fallacious. The charge that the syllogism involves the fallacy of petitio principii is of considerable antiquity. It was advanced by Sextus Empiricus in the second century A.D., and in modern times by Mill and others. Sextus Empiricus held that in every syllogistic reasoning the truth of the conclusion is assumed in the truth of the major premise and therefore the fallacy of petitio principii is committed. The fallacy of petitio principii occurs if the premise from which a conclusion is deduced is itself proved by evidence which includes the conclusion. Sextus Empiricus supposed that the major premise of the syllogism is always an enumerative universal, and that therefore the conclusion cannot go beyond it, as in the syllogism, All the apostles were Jews, Peter was an apostle, therefore Peter was a Jew. It is undeniable that in such a reasoning the conclusion is required to prove the major premise, and therefore is assumed in the major premise. But

The arguments of Sextus Empiricus and Mill as to why syllogistic reasonings involve the fallacy of petitio principii.

here also, we must remember, the conclusion follows not merely from the major premise, but from two premises, the major and the minor. And Sextus Empiricus is wrong in supposing that the universal proposition in the syllogism is always an enumerative universal. We have already found that a syllogism, which is of value, requires a major premise which is a true abstract universal.

**The charge that the syllogism is fallacious refuted.**

Mill and his followers also regarded syllogisms as unnecessary and fallacious. Mill does not seem clear as to the meaning of the fallacy of petitio principii. Does he mean that in a syllogism, if the conclusion is false, the premises (when the process of reasoning is correct) are also false? This is not the proper meaning of the fallacy of petitio principii. In this sense not only the syllogism but every other form of demonstration, geometrical or other, involves petitio principii, since in every demonstration, if the conclusion is false, the premises cannot be true if the process of reasoning is valid. But the charge of petitio principii can only be brought in a true sense against the syllogistic argument, if it can be shown that in syllogism the conclusion is required as evidence to prove the major premise and is thus assumed in that premise. But we have already seen good reason to conclude that this contention cannot be granted.

Keynes and Johnson rightly remark that the validity of syllogistic reasoning rests upon subjective or epistemic conditions, not upon objective or constitutive conditions. Objectively viewed, the syllogism can be regarded as fallacious, because the objects to which the major premise refers include the object or objects to which the conclussion refers; and from this point of view Mill regarded the syllogism as involving the fallacy of petitio principii. But we have already pointed out that the essence of inference is movement of thought from the known to the unknown, and if we do not know the conclusion as soon as we know the premises, and the conclusion imparts some new information, there is inference proper. If so, the epistemic or subjective validity of the syllogism cannot be questioned, and the epistemic factor of inference, which is concerned with what we happen

to know, rather than the constitutive factor, which is concerned with what is thought about, is what determines the validity of inference.

Summary of the results reached.

Welton therefore sums up as follows the arguments required to prove the validity of syllogistic reasoning: "The major is essentially not a mere summation of observed instances; the minor is a necessary part of every syllogism; it is possible to accept premises without drawing the conclusion, and hence to make progress in knowledge by means of syllogism; and the fact of inference depends on the rigidity of the proof, not on its novelty." A syllogism does involve the fallacy of petitio principii if its major premise is an enumerative universal. The syllogism may also be regarded as fallacious if viewed objectively; but in deductive reasoning, as we have found, subjective validity is sufficient.

Range and limitations of the syllogism. Attempt to reduce all inferences to the syllogistic form is not tenable.

We may now consider the range and limitations of the syllogism. It has been claimed by Whately, Bowen. Mansel and others that the syllogism is the only type of mediate inference and that every mediate inference can be syllogistically expressed. Whately attempted to reduce inductive reasoning to the syllogistic form. Mansel attempted to reduce the argumentum a fortiori to the syllogism. Thus he reduces the argument, A is greater than B, B is greater than C, ∴ A is greater than C, to the following syllogism: Whatever is greater than a greater than C is greater than C. Syllogism is only one of the various forms of mediate inference. and should not be regarded as the only type. But it must be admitted that the syllogism is a very important from of mediate inference.

We have seen that the relation from which the syllogistic inference is drawn is that of subject and predicate; but from other relations we can draw inferences which cannot be regarded as syllogistic. Thus Welton says,

Bradley's consideration of the principles of synthesis of relations shows that the syllogism is not the only type of mediate inference.

"The syllogism deals only with propositions which express the relation of subject and attribute, and inferences from other relations, though they may be perfectly valid, not only are not made syllogistically, but cannot be satisfactorily expressed in that form". We require a logic of relatives to find out all the relations from which inferences may be drawn, but since relations are vague, it is difficult to elaborate such a logic in its perfect from. Bradley says that the principles of synthesis of relations, upon which all inferences depend, are as many as there are categories, but he mentions five main types of these principles. These are :—

(1) Synthesis of subject and attribute, as illustrated in the case, All men are mortal, Socrates is a man, ∴ Socrates is mortal. All syllogistic inference rests upon this principle of synthesis.

(2) Synthesis of identity, as illustrated in the case, A is the brother of B, B of C, and C is the sister of D, ∴ A is the brother of D.

(3) Synthesis of degree, as illustrated in the case, A is hotter than B, B is hotter than C. ∴ A is hotter than C; or, A is greater than B, B than C, ∴ A than C. This principle of synthesis is the basis or ground of the argumentum a fortiori.

(4) Synthesis of time, as illustrated in the case, A is before B, and B before C, ∴ A before C; or, A is after B, B is contemporary with C, ∴ A is after C.

(5) Synthesis of space, as illustrated in the case, A is north of B, and B west of C, ∴ C is south-east of A.

The syllogism not the only type of deductive mediate inference, but by far the most important.

Though this list of the principles of synthesis of relations is not exhaustive, it throws much light upon the problem of the different forms of mediate inference. Of these principles, the syllogistic inference rests upon the first alone. The inferences illustrated in the other four cases are not syllogistic, and have no universal major premise. Thus we find that the syllogism is not the only form of mediate inference, for all the inferences illustrated above are deductive, though only the first is syllogistic. Though this is true, it is undeniable that the syllogism is the most important, the most universal, and the most accurate form of inference, and the major portion of our inferences are syllogistic. Though the syllogism is limited in range, it is perfect as far as it goes.

Russell has thrown much light upon the problem of the logic of relatives. It is owing to his work that we can now provide a list of the most important relations which are the ground of inference :—

1. *Subject and predicate relation.*—This relation, as we know, is the ground of syllogistic inference.

2. *Transitive relation.*—This also can yield valid conclusions. "A transitive relation is such that if it relates one term to a second teim and this second term to a third term, then it must relate the first to the third term". *e.g.*, If A is younger than B, and B is younger than C, then A is younger than C.

Russell's classification of relations from which inferences may be drawn.

3. '*An intransitive relation* is such that if it relates one term to a second and this second to a third, it cannot ıelate the first term to the third. Thus if A is the father of B and B is the father of C, A cannot be the father of C."

4. "*A non-transitive relation* is such that it may or may not be transitive. Thus A, who is a friend of B who is a friend of C, may or may not be a friend of C."

5. *Symmetrical relations.*—"A symmetrical relation is such that if it relates one term, A, to another, B, it also relates B to A;" *e.g.*, 'equals', 'is identical with.' If A equals, or is identical with, B, then B equals, or is identical with, A.

6. *Asymmetrical relations.* "An asymmetrical relation is such that if it relates A to B it cannot relate B to A;" thus 'after' is an asymmetrical relation. If A is after B, then B is not after A.

7. *Non-symmetrical relations.*—"A non-symmetrical relation is such that if it relates A to B, it may or may not relate B to A;" *e.g.* 'hate'. If X hates Y, Y may or may not hate X.

8. *Connected relation.*—" ' connected relation is such that given any two of the terms which it relates, it either relates the first to the second or the second to the first, and it may relate both the first to the second and the second to the first. Thus among the points on a line, of any given two, one must be to the left of the other, or this must be to the left of the one." "Relations which are at once transitive, asymmetrical and connected are of special importance in mathematical reasoning, since they give rise to series." Transitiveness is a relation which secures the validity of non-syllogistic reasoning. Some relations occurring in non-syllogistic arguments are asymmetrical as well as transitive. This is evident in the a fortiori reasoning.

The progress of knowledge depends upon correct inference, and we know that the two main types of inference are syllogism and induction. Some logicians, such as Whately, Mansel and others, supposed, as we have found, that the sylogism is the only type of valid mediate inference, and they

refused to regard induction as an independent form of it. On the other hand, Mill and his followers minimised the importance of syllogistic reasoning; they supposed that inference is always from the particular to the particular, that the major premise of the syllogism is a mere memorandum which registers observed instances, and that the passage from the general to the particular in the syllogism is not inference but mere interpretation of what has already been observed. Our discussion has shown that neither syllogistic inference, nor inductive inference, can be ignored by thought. Both types are necessary for knowledge. The major premise of the syllogism, if it is not a self-evident axiom, must express a belief which has been proved by induction, if the syllogistic reasoning is to be valid. Similarly, if induction is to prove or discover some law which is true and useful, it must not depend simply upon observation of facts, but should also be formally valid and its conclusion verified by syllogistic reasoning.

**Dogmatism either of formal logicians or of the advocates of material logic is not tenable.**

# CHAPTER XVI

## FORMAL FALLACIES

The meaning of fallacy.

The term fallacy is used in different senses. A false statement is often said to be a fallacy. Thus we say that the statement that men walk on their heads is fallacious. It is also held that a false belief is a fallacious belief. Thus to believe in ghosts is sometimes regarded as a fallacious belief. Even an ungrammatical sentence, *e.g.*, 'Five men is coming', is said to be a fallacious sentence. But in logic the term fallacy is used to denote an argument that is really false, even though it might appear to be true. Properly speaking, a false argument alone should be regarded as a fallacy. A false belief or an incorrect statement should not be so regarded. According to Joseph, "A *fallacy* is an *argument* which *appears to be conclusive when it is not*; and the chief use of studying fallacies must be that we may learn to avoid them." The fallacy is also defined as "a violation of logical principles disguised under a show of validity." It is therefore clear that not every confusion of thought or prejudice is a fallacy.

The use of the study of fallacies.

Many logicians are of the opinion that fallacies must not be studied as a distinct subject in connection with logic. They point out that the violations of grammatical rules are not studied separately, and in the same way the violations of logical principles need no be studied separately. It is true that a knowledge of the principles of logic enables us, to some extent, to detect fallacies arising from their violation, and a study of fallacies does not enable us to avoid errors in reasoning. It is true that a psychologist can well understand the principles of the operations of the mind only when he studies both ab-

normal and normal mental processes, and a physiologist can gain a clear understanding of the conditions of health only by studying the principles both of disease and of health. But a logician cannot acquire much insight into the principles of valid thinking by studying fallacies. Though this is true, it cannot be denied that we have complete understanding of a thing only when we come to know its opposite as well. So a study of the transgressions of logical principles with the study of the principles themselves may enable us to see more clearly what the principles of valid thinking are. Therefore the study of fallacies cannot be regarded as itself fallacious or useless.

Sources of fallacies.

According to Mill there are two main *sources* of *fallacies, viz., moral* and *intellectual.* Fallacies are often committed owing to passion, indolence, prejudice, etc.; these are the moral sources of fallacies. Again, men commit fallacies because they sometimes fail to think rightly. Fallacies which are due to the failure to think accurately are those which have an intellectual source.

Classifications of fallacies.

It is very difficult to classify fallacies. Various classifications are given but none of them is scientific. Aristotle classifies all fallacies under two main heads, *viz.,* (1) fallacies *in dictione,* under which he includes Equivocation, Amphiboly, Composition, Division, and Figure of Speech; and (2) fallacies *extra dictionem,* under which he includes Accident, Secundum quid, Ignoratio elenchi, Petitio principii, Non causa pro causa, Consequent, and Many Questions. The former class of fallacies, but not the latter, has its source in the ambiguity of language. Whately classifies fallacies under two main heads, *viz.,* (1) logical or formal, which he sub-divides into (*a*) purely logical, (*b*) semi-logical, and (2) material. Mill gives the following classification:

- Fallacies
  - of simple Inspection............1. Fallacies a priori.
  - of Inference
    - From evidence distinctly conceived.
      - Inductive fallacies
        - 2. Fallacies of observation.
        - 3. Fallacies of generalisation.
      - Deductive fallacies
        - 4. Fallacies of ratiocination.
    - From evidence indistinctly conceived ......5. Fallacies of confusion.

Other logicians classify fallacies under three main heads, *viz.*, (1) Non-logical or material fallacies, which are subdivided into (*a*) Premise unduly assumed (petitio principii), (*b*) Irrelevant conclusion or Ignoratio elenchi; (2) Non-inferential logical fallacies, which include fallacies of definition, division, classification, etc.; (3) Inferential fallacies, which are either (*a*) inductive or (*b*) deductive. Deductive inferential fallacies are again subdivided into (i) Formal fallacies and (ii) Semi-logical fallacies. We shall not treat of non-logical or material fallacies and inductive inferential fallacies in this part. Other fallacies, which are connected with the problems treated of in this part, will be discussed under three main heads, (1) *Non-inferential logical* fallacies, (2) *Inferential formal* fallacies, (3) *Semi-logical* fallacies. This classification, though convenient, does not claim to be a scientific one.

### Non-inferential Logical Fallacies

In consequence of failure to understand the meaning of terms clearly, we often use concepts which are vague. If a term is clearly defined and understood, we can get rid of fallacies incidental to *vague conception*. Thus the term 'infinite' is

Vague Conception.

used to mean 'finite at a very extended limit,' though the term 'infinite' really means 'that which is limitless.' Similarly the term 'eternal' is used to mean 'enduring throughout a very long time.' But the term 'eternal' really connotes existence out of time.

**Inconsistent terms and propositions.**

Fallacies incidental to the use of *inconsistent terms* occur when incompatible words are combined into terms. These fallacies also occur because we often fail to see the meaning of words clearly and distinctly. Thus such terms as 'indivisible matter,' 'sweet sorrow,' 'circular straight line,' 'pleasant anger,' 'angular happiness' etc., are inconsistent terms. The use of such terms should be avoided in logic. Again, sometimes we commit the fallacy of *inconsistent propositions*, when the subject and the predicate which are combined in a proposition are inconsistent terms. Thus the propositions, 'Man is immortal,' 'Every rule has an exception,' 'Mind is visible,' 'Truth is consistent falsehood,' 'Misery is desirable,' are inconsistent. The proposition 'Epimenides the Cretan says that all Cretans are liars,' is also self-contradictory or inconsistent because it means that Epimenides can only speak the truth if he lies, and lies if he speaks the truth. We use inconsistent propositions only when the meaning of the terms of a proposition is not well defined and understood.

**Fallacies arising from violation of the rules of Definition and Division.**

We have already discussed, in the chapters on Definition and Division, the fallacies which arise from the violation of the rules of definition and division. So these fallacies need not again be treated of here. Students will find them clearly and fully discussed in the chapters mentioned. The *fallacies of definition* are redundant definition, too wide definition, too narrow definition, obscure definition, tautologous or circular definition, negative definition and figurative definition. The *fallacies of division and classification* are cross or overlapping division, too wide division, too narrow division,

metaphysical division, and physical division. These fallacies, though logical, are not fallacies of inference.

## Inferential Formal Fallacies

Under this head are included those fallacies which arise from the violation of the rules of opposition, of eduction or immediate inference, of syllogisms, of mixed syllogisms, of sorites and of epicheirema. These fallacies have been explained clearly in connection with the discussion of the topics mentioned.

Fallacies to be treated of in this section.

Fallacies incidental to *opposition* occur when the rules of inference by opposition are violated. When two propositions are in contrary opposition, we can pass from the truth of one to the falsity of the other, but not conversely. Thus it is fallacious to argue that if an A proposition is false, the corresponding E proposition is true. Similarly of two subaltern propositions, when the universal is true, the corresponding particular is true, but it is fallacious to argue that when the universal or subalternans is false, the corresponding particular or subaltern is also false. Again, though we can pass from the falsity of the subaltern to the falsity of the subalternans, it is fallacious to argue from the truth of the subaltern to that of the subalternans. Again, of two sub-contrary propositions, if one is false, the other is true, but it is fallacious to argue that when one is true the other is false. The confusion between contrary and contradictory opposition often gives rise to fallacy.

Fallacies incidental to opposition.

Eductions, we have found, are of two kinds, *viz.*, material and formal. We have shown how material eductions may lead to fallacies, and that the correctness of material eductions depends upon the knowledge of facts. Students will find that fallacies incidental to *material eductions* have been treated in connection with the discussion of the

Fallacies incidental to material and formal educations.

forms of those eductions. We need not repeat them here. Fallacies incidental to *formal eductions* are those which arise from the violation of the rules of conversion, obversion, contraposition and inversion. The commonest of these fallacies consists in simply converting an A or an O proposition. If A is simply converted, or O is converted at all, there is usually the fallacy of *distribution.* It is a rule of eduction that no term should be distributed in the conclusion if it has not been distributed in the premise. If A is simply converted the predicate term of the original proposition, which is undistributed there, becomes distributed in the conclusion, as the subject of a universal proposition. Again, if O is converted, the subject term of the original proposition, which is undistributed, becomes distributed in the conclusion, as the predicate of a negative proposition. Fallacies incidental to contraposition and inversion also occur when a term which is not distributed in the premise becomes distributed in the conclusion. These fallacies are really fallacies of conversion. Thus if we conclude from 'No S is P' to 'All not-P is S,' there is the fallacy of *contraposition.* To infer from 'Every S is P' that 'No not-S is P,' or from 'No S is P' that 'Every not-S is P,' would be to commit the fallacy of *illicit inversion.* Similarly if we pass from the proposition 'Thought is existent' to the proposition 'What contains no element of thought is non-existent,' we commit the fallacy of illicit inversion. Students may consult the discussion of formal eductions if they wish to understand clearly the fallacies which arise from the violation of the rules of formal eduction.

Fallacies incidental to syllogisms.

The fallacies which arise from the violation of the rules of syllogism have been fully discussed in the chapter on syllogism. Here we need only name them. They are—(1) the fallacy of *four terms* (Quaternio Terminorum), (2) the fallacy of *undistributed middle,* (3) the fallacy of *illicit process* of the *major* term or illicit major, (4) the fallacy of

*illicit process* of the *minor* term or illicit minor, (5) the fallacy of *negative premises.* Fallacies are *abstract* when they are committed openly, and *concrete* when they are hidden by the language. The fallacy of undistributed middle is very often committed, and should be guarded against.

Fallacies incidental to mixed syllogisms have been fully discussed in the chapter on mixed syllogisms. Fallacies of mixed *hypothetical* syllogisms are two in number, *viz.*, (1) *denying the consequent,* (2) *affirming the antecedent.* The former occurs when by denying the antecedent we deny the consequent, and the latter occurs when by affirming the consequent we affirm the antecedent. These fallacies are called by Aristotle fallacia consequentis.

Fallacies incidental to hypothetical syllogisms.

Fallacies occur in mixed disjunctive syllogisms when from a disjunctive proposition whose alternatives are exclusive but not exhaustive we affirm one of the alternatives by denying the other. Again, when a disjunctive proposition has its alternatives exhaustive but not exclusive, we commit a fallacy if we deny one of them by affirming the other. (For illustrations see the chapter on Mixed Syllogisms).

Fallacies incidental to disjunctive syllogisms.

Dilemmatic arguments are fallacious when the alternatives are not exclusive and exhaustive. We have seen, in discussing the dilemma, how it is possible to escape between the horns of a dilemma, to take it by the horns, or to rebut it. We have pointed out in what circumstances dilemmatic arguments are fallacious. (Students are referred to the chapter on Mixed Syllogisms to find the fallacies incidental to the dilemma).

Fallacies incidental to dilemmas.

Fallacies also arise from the violation of the rules of sorites

and epicheirema. Students are referred to the chapter on trains of syllogism for the rules of sorites and epicheirema, and for the fallacies which may arise from their violation. Fallacies incidental to enthymemes are nothing other than the fallacies of syllogisms. To find out the fallacy in an enthymematic argument we should reinstate the suppressed proposition, and state the syllogism in full. Sophists often have recourse to enthymemes to delude their opponents.

Fallacies incidental to sorites, epicheirema, and enthymemes.

We may here name another class of fallacies known as the fallacy of *Many Questions* (Plures Interrogationes). It occurs when only one answer is demanded to several questions contained in one sentence. Thus if anyone asks, "Have you left off beating your father" no single answer. whether 'yes' or 'no', can be given. If one answers 'yes,' it implies that one used to beat his father; if the answer is 'no,' it implies that one beats one's father even now. Similarly it frequently happens that no single answer can be given to such questions as, "Where did you hide the goods you stole last night?" "Have you given up drinking?" "Have you cast your horns?" etc. Sophists often asked such questions to place their opponents in a difficulty. Lawyers often use such questions in cross-examination. In inductive reasoning we often commit a fallacy when we suppose that a cause which produces an effect is the only cause of the event. Thus when we suppose that death has but one cause, or that a particular disease has but one explanation, or that hunger can be appeased only by one kind of food, we commit such a fallacy.

The fallacy of Many Questions (Plures Interrogationes)

### Semi-logical Fallacies

Under this head we would discuss all those fallacies which arise from the ambiguity of language. Some of these are

Semi-logical fallacies arise from the ambiguity of language.

really fallacies of definition, for when terms are not clearly defined they are often used ambiguously. Other fallacies of ambiguity are due to the ambiguous construction of propositions. Fallacies which are due to the ambiguity of language are called by Aristotle fallacies *in dictione*. They are Equivocation, Composition, Division, Figure of speech, Amphiboly and Accent. Besides these, Accident and Secundum quid, which Aristotle includes under the class of fallacies extra dictionem, are also regarded as fallacies due to the ambiguity of language. All these fallacies, and some others, are included by Mill in 'fallacies of confusion'. Let us now explain these fallacies one by one.

Fallacy of Equivocation.

1. *Aequivocatio* or *Homonymia*.—Equivocation occurs through the use of words capable of two or more meanings. We use equivocal terms only when we fail to define them. In a syllogistic argument, the middle term or one of the extremes may be ambiguous, and then we have the fallacy of four terms. The following argument is an example given by the old logicians, in which the middle term is ambiguous:—"The end of a thing is its perfection; death is the end of life; therefore death is the perfection of life." 'End' in the major premise means 'aim' or 'ideal,' and in the minor premise it means 'termination'. The following is another example to illustrate the fallacy of ambiguous middle: "Knowledge is power; perception is knowledge; therefore perception is power." Here the middle term 'knowledge' is used ambiguously in the premises. In one case 'knowledge' means knowledge in general, in the other it means a particular species of it. Words often change their meaning, and owing to the use of such words the fallacy of equivocation often occurs. Thus, 'publication' once meant communication to others, but 'to publish' now means to bring out something in print. So 'utter' meant simply to give out, but it now means to give out of the mouth

in words. The meaning of a word should be decided by its current use, not by its etymological sense. The term 'scarcity of money' is often used ambiguously. It may mean either scarcity of currency or scarcity of capital. 'Government' may mean the system of laws, or the persons who are entrusted with the task of enforcing those laws. 'Nature' may mean physical nature, instinct, reason, etc. Puns are logically instances of this type of fallacy. Joseph gives the following example of equivocation: "Men who have recovered are well, the sick man has recovered, therefore the sick man is well." Here the minor term is ambiguous. In the minor premise 'the sick man' means the man who *was* sick, in the conclusion it means the man who *is* sick. The following is an example in which the major term is ambiguous or equivocal (ambiguous major): 'No courageous creature flies; the eagle is a courageous creature; therefore the eagle does not fly.' Here the major term 'fly' in the major premise means to flee, while in the conclusion it means to move through the air with wings.

2. The Fallacy of *Figure of speech* (Figura Dictionis or Sophism).—This consists in supposing that words which are similar in form are similar in meaning. It is wrong to suppose that poeta ('a poet') is in the feminine gender because most Latin words with the same termination are so. Mill is wrong in arguing that because what people actually hear is audible, and what people actually see is visible, therefore what people actually desire is desirable. 'Desirable' means that which ought to be desired, not what is actually desired. The meaning of 'desirable' is not parallel to the meaning of the words 'audible' and 'visible'. The most important falacies of this class arise from the use of paronymous terms or conjugate words. Fallacies of figure of speech are often called fallacies of paronymous terms. Different parts of speech derived from the same root are not always similar in meaning. Thus artist, artisan, artful are not similar in

The fallacy of Figure of Speech (Figura Dictionis).

meaning. Similarly pity and pitiful, presume and presumption, apprehend and apprehension, design and designer, image and imaginary, faith and faithful, are not similar in meaning, though each pair of these words is derived from the same root. What is imaginary is unreal, but the image of an object, whether of a flower or of a stone or of a house, is real. An artful man need not be an artist. To apprehend means to perceive or understand, but apprehension often means fear or dread. These fallacies also are offences against definition. When paronymous terms are used in syllogisms, we have the fallacy of four terms. Mill gives the following examples of such fallacies:—Murder should be punished with death; This man is a murderer; therefore He deserves to die. Here we proceed on the assumption that to commit murder and to be a murderer who deserves death are equivalent expressions. Projectors are unfit to be trusted; This man has formed a project; therefore He is unfit to be trusted. A sophist will try to make men believe that a projector is simply one who forms a project, but 'projector' also has the meaning of a promoter of bogus companies Welton and Monahan give the following example to illustrate the fallacy of figure of speech: What a man walks on he tramples on; This man walks on the whole day; therefore He tramples on the whole day. The expression 'walks on' does not mean the same thing in the two premises.

3. The fallacy of *Accident* (Secundum Quid).—Aristotle made a distinction between the fallacy of *Accident* and *Secundum Quid*. But Secundum Quid is regarded by modern logicians as not different from the fallacy of Accident. The following examples illustrate the fallacy of Accident in the Aristotelian sense (fallacia accidentis), which arises when a predication which can be correctly made of a given subject is made of any of the 'accidents' of that subject: (*a*) This dog is yours, This dog is a father. Therefore, this dog is your father. (*b*) Do you know Coriscus. Yes. Do you

The fallacy of Accident explained and illustrated.

know the man approaching you with his face muffled? No. But he is Coriscus, and you said you knew him. (A man approaching with his face muffled need not be Coriscus, because it is an accidental circumstance.) (*c*) Six is few, and thirty-six is six times six, therefore thirty-six is few.

The three forms of the fallacy of Accident or Secundum Quid

We have already remarked that what Aristotelians called Secundum Quid is now called the fallacy of Accident. Such fallacies may have three forms:—(*a*) We may pass from an abstract general rule to a particular concrete case. The fallacy is then technically known as 'fallacia *a dicto simpliciter ad dictum secundum quid.*' The following are examples of this fallacy: Employment of labour is beneficial to the community, therefore unemployed workmen may wisely be set to do work of an entirely useless character, merely to find them employment. Similarly to pass from 'What man has done, man can do,' to the conclusion that 'What Newton or Shakespeare has done you can do,' illustrates this fallacy. From the proverb 'a rolling stone gathers no moss', if we conclude that every commercial traveller must starve, we commit this fallacy. The following example also illustrates this fallacy: Water is liquid, Ice is water, therefore Ice is liquid. In the minor premise 'water' means condensed water. (*b*) We may reason from a concrete special case to an abstract rule. The fallacy is then technically known as 'fallacia *a dicto secundum quid ad dictum simpliciter.*' The following example is given by Welton and Monahan: What you bought yesterday you ate to-day; you bought raw meat yesterday, therefore you ate raw meat to-day. It is not made clear whether rawness is regarded in the major premise as a relevant circumstance, but it is assumed to be relevant in the conclusion. If, finding that a paricular Indian is dishonest, we conclude that all Indians are dishonest, we commit this fallacy. (*c*) We may argue from one special case to another special case. The fallacy is then technically known as 'fallacia

*a dicto secumdum quid ad dictum secundum alterum quid.* To argue from the assertion that to take life in sport is cruel, to the conclusion that to eat flesh from which life has been taken by others is to show a cruel disposition, illustrates this fallacy. The following example also illustrates this fallacy: To inflict pain on another is wrong; the surgeon in performing an operation inflicts pain on another; therefore the surgeon does something wrong. A master asked his servant to roast a stork. The servant was prevailed upon by his sweetheart to cut off one of its legs and give it to her. When the master at dinner-time asked what had become of one of the legs of the stork, the servant answered that storks had but one leg. The master, desiring to confute the servant before punishing him, took him to a place where many storks were standing. They were all standing on one leg, as they are accustomed to do. The master shouted, and they put down their other leg and flew away. On this the servant said, "You did not shout at the dinner table; if you had done so, the roasted stork would have put down its other leg." In this case the fallacy consists in the fact that the servant argued that what was true of a living stork was also true of a roasted stork. This is clearly a fallacy of accident.

4. The fallacy of *Composition* and *Division* (Compositio and Divisio). The fallacy of composition and division occurs when we join together things which ought to be kept separate or when we separate those that are to be kept conjoined. In other words, the fallacy of composition occurs when the same term is distributive in the premises and collective in the conclusion, and the fallacy of division occurs when the same term is collective in the premises but is distributive in the conclusion. These fallacies are also committed when the middle term is taken collectively in one of the premises and distributively in the other. Thus the fallacy of composition and the fallacy of division are the converse of

The meaning of the fallacy of Composition and Division.

each other. Aristotle gives the following example of the fallacy of composition: Two and three are even and odd; two and three are five; therefore five is even and odd. The following are examples of the fallacy of division, in which the middle term is collective in the major premise but distributive in the minor:—All the books on the table weigh ten tons, This is a book on the table, therefore This book weighs ten tons. All the angles of a triangle are equal to two right angles, ABC is an angle of a triangle, therefore ABC is equal to two right angles. When a miser argues that because he cannot buy these books and that watch and this table and that picture all at once, therefore he cannot buy any of them, he commits the fallacy of division. Thus we find that failure to distinguish between disjunctive and conjunctive propositions may give rise to the fallacy either of composition or of division. The following argument also illustrates the fallacy of division:—Three and five are (together) four and four (together); but neither three nor five is four; therefore three and five (together) are not four and four (together).

The fallacy of Division illustrated.

As a result of ambiguity in the construction of propositions, there may be two classes of fallacies (recognised by Aristotle), *viz.* (1) Amphiboly (amphibolia), (2) the fallacy of Accent (accentus). The fallacies treated of in the previous paragraph arise from the ambiguous use of terms, while the fallacies of amphiboly and accent are incidental to the ambiguous construction of propositions.

Fallacies incidental to ambiguous construction of propositions.

1. *Amphiboly* (Amphibolia).—This fallacy is committed when a proposition becomes liable to misinterpretation through its ambiguous construction. The following oracle given to Pyrrhus is an example in point: "Pyrrhus the Romans shall, I say, subdue." This may mean either that

The fallacy of Amphiboly explained and illustrated.

Pyrrhus shall subdue the Romans or that the Romans shall subdue Pyrrhus. The following is another classical example of this fallacy: "The Duke yet lives that Henry shall depose" (witch's prophecy in *King Henry VI*). This proposition may mean either that the Duke shall depose Henry, or that Henry shall depose the Duke. The following line of W. R. Spencer furnishes another good example of this fallacy: "The noble hound the wolf hath slain." This fallacy occurs in modern English chiefly when words are not used in their proper order. The following are examples: "The first photograph is that of a 14-pound pike taken in a back-yard from the top of a step-ladder;" "a lady (through circumstances) wishes to let part of her well-furnished house;" "the Territorial band played the hymns as well as the church organ." These fallacies can be got rid of if sentences are properly constructed.

The fallacy of Accent explained and illustrated.

2. The *fallacy of Accent* (Accentus).—In Greek the same word, if differently accentuated, had a different meaning. So the fallacy of accent was recognised by Aristotle. Sometimes the meaning of a proposition becomes distorted when emphasis is laid wrongly upon some word which should not be emphasised. Thus the proposition, 'Thou shalt not bear false witness against thy neighbour,' may mean that you may bear false witness against those who are not your neighbour, if emphasis is laid upon the word 'neighbour'. De Morgan rightly points out that accents, gesture and manner often make the difference between irony, sarcasm and ordinary assertion. The father spoke to his sons saying, "Saddle me the ass," and the sons saddled the father. (I Kings 13, 27). We need not cite more examples of this fallacy.

# EXERCISES

## INTRODUCTION

### DEFINITION, NATURE, AND SCOPE OF LOGIC

1. What is the primary condition of knowledge? In what sense is knowledge progressive?

2. What is knowledge? State explain, and illustrate its different forms. With which of these forms is Logic concerned, and why?

[**Hints.**—Knowledge involves four factors: (1) a system of *ideas* in the mind, (2) a system of *things* and relations, (3) *correspondence* between these two systems, and (4) a *belief* in their correspondence.]

3. Explain the distinction between perceptual and conceptual knowledge, illustrating your answer with suitable examples.

4. What primary condition should our thought fulfil in order to be true?

[**Hints.**—Truth is ordinarily said to consist in correspondence of ideas to things and relations.]

5. Distinguish between Immediate and Mediate knowledge, illustrating your answer with examples. With which of them is Logic concerned, and why?

6. What are the principal sources of knowledge? Explain the nature of each, and illustrate your answer with examples.

[**Hints.**—The sources of knowledge ordinarily recognised are: (1) *Perception*, (2) *Inference*, and (3) *Authority*. Perception is said to be immediate, while Inference and authority are supposed to be mediate.]

7. As you go out in the morning, you find the roads wet and muddy, and at once come to believe that there has been rain during the night. Say what is immediate and what is mediate in the knowledge you thus come to have.

8. Distinguish between Immediate and Mediate Truth.

[**Hints.**—Knowledge is apprehension of truth. It is immediate or mediate, but truth as such is neither, unless by truth we mean valid knowledge. The real distinction is between immediate and mediate knowledge and not between immediate and mediate truth.]

9. What is thought? Distinguish between the matter and form of thought.

[**Hints.**—Thought is viewed in Psychology as a *process;* in Logic, both as a process and *product.* The thought process consists in such acts as perceiving objects, forming concepts, judging things, reasoning about them, and doing things of a like nature. These mental acts are called *perception, conception, judgment, reasoning,* etc. Their products are called *percepts, concepts, judgments, inferences,* etc. By thought Logic means mainly these products of thinking. It takes only a secondary interest in the process.

By the *matter* of thought is meant the things thought about. In the judgment 'Man is mortal,' the matter consists of the concepts 'man' and 'mortal.' By the *form* of thought is meant the way in which we think about them. In the judgment 'Man is mortal,' we think in terms of affirmation, that is to say, we affirm the predicate 'mortal' of the subject 'man.' In the judgment 'Man is not immortal,' we think in terms of denial; we deny the predicate 'immortal' of the subject 'man'.]

10. Distinguish between the logical and psychological treatment of thought.

11. Distinguish between Formal and Material truth. Which of them constitutes the proper subject-matter of Logic?

12. Distinguish between the consistency of thought and the truth of thought. Discuss the view that Logic is the science, not of truth, but of consistency.

13. Explain and illustrate the distinction between Formal and Material Logic.

14. Explain and illustrate the distinction between Deductive and Inductive Logic.

15. Precisely state the connexion, if any, between (*a*) Formal and Material Logic, and (*b*) Deductive and Inductive Logic.

16. Clearly explain the nature of scientific knowledge, and distinguish it from popular knowledge.

17. Distinguish between Science and Art. Is Logic a Science or an Art?

18. Distinguish between Positive and Normative Science. Is Logic a Positive or a Normative Science?

19. Distinguish between Normative and Practical Science. Is Logic a Normative or a Practical Science?

20. State and explain the definition of Logic which appears to you to be the most satisfactory.

21. Examine the following definitions of Logic:

(*a*) Logic is the art of reasoning.
(*b*) Logic is the science of correct or valid thought.
(*c*) Logic is the science and art of reasoning.
(*d*) Logic is the science of the regulative laws of human knowledge.

(*e*) Logic is the science of the operations of the understanding in the pursuit of truth.

(*f*) Logic is the science of argumentation.
(*g*) Logic is the science of reasoning.
(*h*) Logic is the science of the principles which regulate valid thought.

(*i*) Logic is the science of the regulative principles of thought.

(*j*) Logic is the science of the operations of the understanding that are subservient to the estimation of evidence.

22. What are the functions of Language in relation to Thought? How far is it correct to say that Logic is concerned with language?

23. What are the ways in which language helps thinking?

24. Explain what is meant by defining Logic as 'the science of sciences.'

25. Discuss the relation of Logic to the special sciences.

26. What is the relation of Logic to Psychology?

27. Explain the relation of Logic to Metaphysics.

28. What is the relation between Logic, Ethics and Aesthetics?

29. Why is Logic supposed to be specially related to Psychology, Metaphysics, and Grammar?

30. State and explain the Fundamental Laws of thought.

31. Are all the Fundamental Laws of thought equally fundamental?

32. Why are the postulates of thought, *i.e.*, the Fundamental Laws of thought regarded as fundamental, necessary, formal, and *a priori*?

33. Clearly explain the relation which exists between the three Fundamental Laws of thought.

34. Clearly state and explain the Principle of Sufficient Reason, Can it be regarded as a postulate of thought?

35. State and explain the 'postulate of logic' given by Hamilton.

36. Clearly indicate the province or scope of Logic.

[**Hints.**—Logic is said to underlie all the sciences and arts.

It even underlies Metaphysics. Hence its scope is as wide as wide can be. But it is mainly concerned with inference. So it is as narrow in scope as any other science or art. Inference, however. may be deductive or inductive. So both deduction and induction fall within the scope of Logic. Incidentally, it deals with a good many things subsidiary to inference such as Perception, Conception, Judgment, Definition, Division, Classification, etc., etc. But its primary interest is not in these processes of the understanding. but in their products, such as concepts, judgments, inferences, etc. It is a science of proof as well as of discovery. It is both formal and material, as truth is ordinarily supposed to be. It is concerned not only with thought, but also with language in so far as language is the vehicle of thought ]

37. What is the subject-matter of the science of Logic? Show how it differs from other sciences.

38. Explain the different views that have been held as to the nature and existence of concepts.

39. Explain Realism. Conceptualism, and Nominalism as schools of Logic.

40. What benefits does one expect to derive from the study of Logic?

41. Men often argue correctly without studying Logic. Is this a reasonable objection to the study of Logic?

42. Does Logic make one an adept in reasoning?

# CHAPTER I

## Terms

### Exercises Worked Out

[**Hints.**—When called upon to describe the logical character of a term, why do we say that it is—

(*a*) Simple or Composite;
(*b*) Univocal or Equivocal;
(*c*) Singular or General;
(*d*) Collective or Non-collective;
(*e*) Definite or Indefinite;
(*f*) Concrete or Abstract;
(*g*) Positive, Negative, or Privative;
(*h*) Absolute or Relative;
(*i*) Connotative or Non-connotative.]

1. *Man*—Simple (*i.e.*, single-worded), Equivocal (for it might mean a human being as in 'There's a man at the door,' or mankind as in 'Man is mortal,' or manly spirit as in 'Play the man'), General (but if it means mankind, it is singular), Non-collective (but may be collective if used to mean mankind), Indefinite (but may be definite if it means all men), Concrete (but may be abstract if it implies manly spirit), Positive, Absolute, Connotative.

2. *Whiteness*—Simple, Univocal, Singular, Non-collective, Definite, Abstract, Positive, Absolute, Non-connotative.

3. *The sun*—Composite, Univocal, Singular, Non-collective, Definite, Concrete, Positive, Absolute, Connotative.

4. *A do-nothing fellow*—Composite, Univocal, Particular, Non-collective, Indefinite, Concrete, Privative, Absolute, Connotative.

5. *The Present Prime Minister of the Indian Union*—Composite, Univocal. Singular, Non-collective, Definite, Concrete, Positive, Absolute, Connotative.

**N.B.** It ought to be clear, however, from the above description of terms that, words torn from their contexts cannot be properly described in the logical way. A word or term that is to be so described must form part of a sentence or proposition.

6. *The rose is a very beautiful flower.*—This is a sentence that fulfils all the conditions of a logical proposition. It contains two terms—'the rose' and 'a very beautiful flower.'

*The rose*—Composite, Univocal, General, Non-collective, Definite, Concrete, Positive, Absolute, Connotative.

*A very beautiful flower*—Composite, Univocal, Particular, Non-collective, Indefinite, Concrete, Positive, Absolute, Connotative.

7. *The man is out of his mind.*—It is a proposition containing two terms—'the man' and 'out of his mind.'

*The man*—Composite, Univocal, Particular, Non-collective, Definite, Concrete, Positive, Absolute, Connotative.

*Out of his mind*—Composite, Univocal, Particular, Non-collective, Indefinite, Concrete, Privative (as it implies the possibility of being restored to sanity), Absolute, Connotative.

## Exercises

1. What do you understand by the statement that knowledge has an object?

2. Explain clearly the relation between judgment, proposition and sentence.

3. Analyse the different parts of a proposition. Is the copula a third term in a proposition?

4. Distinguish between words and terms.

5. What are categorematic and syncategorematic words? What are acategorematic words? What is their relation to terms as such?

[**Hints.**—Acategorematic words are not recognised by all logicians. By an acategorematic word is meant a word that can never be used as a term, no matter whether it is used singly or as a part of a term. In fact, an acategorematic word always stands alone; it cannot be joined with any other word in a sentence or proposition. Interjections are typical acategorematic words, e.g., Alas! Ah! etc.]

6. Distinguish between Ideas, Concepts and Terms.

7. Distinguish between names and terms.

8. Explain and illustrate the following:

(*a*) Connotative and Non-connotative terms;
(*b*) Absolute and Relative terms;
(*c*) Negative and Privative terms;
(*d*) Positive and Negative terms;
(*e*) Infinite terms;
(*f*) Abstract, Concrete, and Attributive terms;
(*g*) Singular and General terms;
(*h*) Collective and Distributive terms;
(*i*) Contrary and Contradictory terms.

9. Explain the nature of a proper name, and distinguish it from significant singular names.

10. What are the points of similarity and difference between general names and uniquely descriptive names? How can a general name be transformed into a singular name? Explain by means of examples.

11. Define collective and distributive terms and explain their nature with examples. Can collective names be both singular and general? Illustrate your answer.

12. Why is it held that the real distinction is not between collective and distributive names but between the collective and distributive use of names? Explain with illustrations.

13. Explain the nature of substantial terms and give some examples.

14. Define concrete and abstract terms and explain their nature by illustrations.

15. Why is it said that a real distinction exists not between concrete and abstract terms but between the concrete and abstract use of terms?

16. What are the different kinds of abstract names? Explain by illustrations.

17. Explain with examples what is meant by saying that concrete and abstract terms go in pairs.

18. Can abstract terms be both singular and general? Explain with examples.

19. Define positive and negative terms and explain their nature with examples.

20. What is a privative term and how is it related to positive and negative terms? Explain with examples.

21. When are terms incompatible? Explain with examples the nature of contrary and contradictory terms.

22. Why are negative terms regarded as indefinite and infinite? Explain with illustrations. Are negative terms significant? Are negative terms logically useless?

23. Show by illustrations that some terms are negative in appearance but positive in meaning.

24. Why is it said that contradictory terms are both exhaustive and exclusive while contrary terms are only exclusive and not exhaustive?

25. Define absolute and relative terms and explain their nature with illustrations.

26. Give examples to show that correlative terms have a common ground. What is meant by 'fundamentum relationis'?

27. Are all relative terms general?

28. Are all terms relative? Is there any absolute term?

29. Define particular and universal terms. Explain their nature with examples.

30. Define univocal, equivocal and analogous terms.. Explain their natuie with examples.

# CHAPTER II

## Intension and Extension, Connotation and Denotation, Connotative and non-Connotative terms

### Exercises

1. Explain what is meant by saying that a term has both subjective and objective aspects.

2. Explain clearly with examples the meaning of denotation and connotation.

3. What are the different meanings of intension? Is the meaning of intension and connotation the same?

4. Why is connotation regarded as the same as conventional intension?

5. Distinguish between extension, denotation and exemplification.

6. What is meant by saying that the denotation and connotation of a term vary in inverse ratio? Do they really vary in inverse ratio in all cases? Explain with examples.

7. What are the limits of inverse variation of denotation and connotation of a term?

8. Define connotative and non-connotative terms, and explain their nature with examples.

9. Are proper names connotative? Give some examples of non-connotative terms and explain why they are non-connotative.

10. What classes of terms are connotative, and why? Explain with illustrations.

11. Can you give examples of terms that have—
    (*a*) both denotation and connotation;
    (*b*) only denotation but no connotation;
    (*c*) only connotation but no denotation;
    (*d*) neither connotation nor denotation?

[**Hints.**—Such terms as 'a round square', 'four-sided triangle', etc., are said to have neither denotation nor connotation. But these ought not to be regarded as terms at all.]

12. Name the classes of terms that are connotative and those that are non-connotative.

# CHAPTER III

## The Categories of Aristotle and the Doctrine of Predicables, Verbal or Analytic, Real or Synthetic, and Formal Judgments.

### Exercises Worked Out

1. *Whales are mammals.*—The subject 'whales' is a species in relation to the genus 'mammals'.

**N.B.** But this is pure logic. In natural history things are not so simple as this. There a kingdom is divided into classes, a class into sub-classes, a sub-class into orders, an order into sub-orders, a sub-order into families, a family into sub-families, a sub-family into genus, a genus into sub-genus, a sub-genus into species, a species into sub-species, a sub-species into varieties, a variety into sub-varieties, until at last we come down to the individual members comprising the whole kingdom.

2. *Man is the only animal that uses fire.*—Here the predicate 'the only animal that uses fire' is an accident in relation to the subject 'man'. Since all men are fire-using animals, the attribute in question is an inseparable accident of the class 'man'.

3. *Man is the only animal that knows how to put two and two together.*—Here the predicate 'the only animal that

knows how to put two and two together' is a property (proprium) of the subject 'man', as the capacity for drawing conclusions from data follows from the connotation of 'man', —from his rationality, in fact

4. *Truth is veracity in speech and act.*—Here the subject 'truth' and the predicate 'veracity in speech and act' are synonymous terms that can have no relation of dependence between them.

5. *His heart is in the right place.*—The predicate 'in the right place' is a separable accident in relation to the subject 'his heart'. It is a separable accident, not of a class, but of an individual. which, in this case, is 'his heart'.

## Exercises

1. How does Aristotle classify categories? Explain the Aristotelian categories with examples.

2. Are the Aristotelian categories a classification of terms or of objects or of concepts?

3. Distinguish between categories and predicables.

4. Clearly explain the nature of predicables, with examples, and bring out their distinction from predicates as such.

5. State and explain the list of predicables given by Porphyry. Illustrate your answer.

6. Explain the following terms:—

Genus, Species, Summum Genus, Infima Species, Subaltern Genus, Subaltern Species, Cognate or Co-ordinate Species, Proximate or Proximum Genus, Cognate Genus, Real Kind, Real Essence.

7. What is Porphyry's tree? Draw and explain it.

8. Define verbal, real and formal judgment. Explain them with examples

9. Why is verbal judgment called analytic, and real judgment synthetic? Is the distinction between analytic and synthetic judgments sound?

10. Distinguish between analytic and synthetic propositions with reference to genus, definition, species, differentia, proprium and accidens.

# CHAPTER IV

## The Problem of Definition

### Exercises Worked Out

1. *Oxygen is a gas.*—Too wide definition. It states something less than the connotation of oxygen, and so fails to distinguish it from other gases such as nitrogen, hydrogen, carbon di-oxide, etc.

2. *A gentleman is a person who moves in good society.*—An accidental definition, better called a description.

3. *A poet is an apostle of sweetness and light.*—A figurative definition; may be called a description, but quite obscure even for a scientific description.

4. *A net is a reticulated fabric decussated at regular intervals.*—An obscure definition that, moreover, moves in a circle, since 'a net' and 'a reticulated fabric' are synonymous terms.

5. *Pleasure is the absence of pain.*—A negative definition that fails even to recognise that pleasure as a positive feeling does not consist in a mere absence of pain.

6. *Man is a civilised creature.*—If 'civilised' means also rational, the definition is too narrow, as it leaves out the so-called 'uncivilised' races of mankind. The word 'creature' is

ambiguous. Ordinarily it has a wider denotation than the word 'animal', although the two words are often used synonymously. Anyway, the definition is vague.

7. *A triangle is a plane figure, bounded by three straight lines, and having three angles that are together equal to two right angles.*—A redundant definition. It overstates the connotation by bringing into it a property (actually two properties) that follows from the connotation as a matter of course.

## Exercises

1. Explain the general nature of Logical Definition, making explicit the point whether definition is of names or of things.

2. Explain with examples nominal. real, substantial and genetic definition. Explain the meaning of the statement—"Definition should be *per genus et differentiam.*"

3. Explain with examples ostensive, biverbial, extensive, descriptive and analytic definition.

4. What are the limits of definition?

5. State and explain the rules of definition. Name and exemplify the fallacies which arise from their violation.

6. Explain with examples—circular, redundant, too wide, too narrow, figurative, obscure, negative and tautologous definition. Are these definitions valid? If so, how? If not, why not?

7. What should we do when a scientific definition of a term is not possible?

8. Determine the relation of Definition to the Predicables.

9. Distinguish between Definition and Description.

10. Examine the following definitions:—
   (*a*) A pharmacy is a drug store.
   (*b*) Law is nothing but common sense.
   (*c*) Knowledge is power.

(*d*) Mind is a thinking substance.
(*e*) Life is the continuous adjustment of inner to outer relations.
(*f*) Banks are institutions indispensable for commerce.
(*g*) Causality is uniform antecedence in time.
(*h*) Architecture is frozen music.

## CHAPTER V

### The Doctrine of Division

#### Exercises Worked Out

1. *A plant is divided into stem, root, branches, and leaves.*—A case of physical division or partition. The name 'plant' is not applicable to these constituent parts of the plant. A plant is an individual thing.

2. *Mind has been divided by psychologists into thinking, feeling, and willing.*—A case of metaphysical division or conceptual analysis. Mind is also an individual thing or substance, and not a class.

3. *Indians are divided into rich, poor, malarious, and consumptive.*—A case of cross division. It makes use of more than one principle of division at the same time. It is also a case of overlapping division, as the sub-classes do not exclude one another.

4. *Sciences are divided into physical, moral, and medical.*—Too narrow division. The sub-classes taken together are not co-extensive with the class divided.

5. *Men are often said to be either knaves or fools.*—Too narrow division. There are men who belong to neither of the categories.

## Exercises

1. Explain the nature of Logical Division and determine its relation to Definition.

2. Distinguish between logical division, metaphysical division, physical division and verbal division. Give examples. What are the limits of division?

3. Explain what is meant by saying that division should be progressive.

4. Does division require a knowledge of facts?

5. Explain with examples the rules of division. Name and exemplify the fallacies which arise from their violation.

6. Explain with examples cross, overlapping, too wide and too narrow division.

7. What is division by dichotomy? Is this division satisfactory? State arguments in favour of, and against, this mode of division.

8. Test the following divisions:—

   (*a*) Sciences into Theoretical, Practical, and Normative.
   (*b*) Human beings into men, women, and children.
   (*c*) Material bodies into Solid, Liquid, Heavy, and Light.
   (*d*) Colleges into Science, Arts, and Law Colleges.
   (*e*) Books into good, expensive, and worthless.
   (*f*) Men into knaves and fools.
   (*g*) 'Fish, fowl, and good red herring.'
   (*h*) A room into roof, floor, walls, and ceiling.
   (*i*) Mind into Thinking, Feeling, and Willing.
   (*j*) Trains into local and electric.

## CHAPTER VI

### The Definition and Nature of Proposition

1. Define judgment and proposition, and clearly explain the relation between them.

2. Distinguish between a judgment, a proposition, and a sentence.

3. What is meant by saying that every proposition has both subjective and objective aspects? In what does a proposition differ from a judgment?

4. Distinguish between the grammatical, metaphysical and logical subjects of a proposition.

5. In what sense is a judgment necessary, universal and constructive? Explain the statement that what is once true is always true.

6. Explain the logical character and function of the copula.

7. Does the copula always imply existence?

8. Is the sentence the same as the proposition? Can there be any proposition with one term only?

# CHAPTERS VII-VIII

## Forms of Proposition & Diagrammatic Representation of Propositions

### Exercises Worked Out

[**Hints.**—When called upon to describe the logical character of a proposition, say whether it is—

(*a*) Simple or Compound ;
(*b*) Categorical or conditional :
(*c*) Affirmative or Negative ;
(*d*) Universal or Particular ;
(*e*) Necessary, Assertory, or Problematic ;
(*f*) Verbal or Real.

If it is a conditional proposition, say whether it is—

(*g*) Hypothetical or Disjunctive.

In reducing a proposition to its proper logical form, always go by the meaning, and not by the form or grammatical construction. Resolve a compound proposition into its constituent simple propositions. The copula must always be some form of the verb 'to be' in the present tense, if you are to tread in the footsteps of the orthodox logicians. To be sure, that's the safest course to follow.]

1. *God is.*—Its logical form would be *'God is existent.'* It is simple, categorical, affirmative, universal, assertory, real. An A proposition.

2. *Not all men are happy.*—Logical form: *Some men are not happy.* It is simple, categorical, negative, particular, assertory, real. An O proposition.

3. *Few persons can keep a secret.*—Logical form: *Some persons are not those that can keep a secret.* It is simple,

categorical, negative, particular, assertory, real. An O proposition.

4. *Two and two make four.*—Logical form: *All combinations of two and two must be four.* It is simple, categorical, affirmative, universal, necessary, verbal. An A proposition.

5. *John as well as James deserves praise.*—This is a compound proposition made up of the following two simple ones:

(*i*) *John is deserving of praise.*
(*ii*) *James is deserving of praise.*

These two simple propositions are of the same nature. They are categorical, affirmative. universal (the subject in both the cases being definite), assertory, real. Both are A propositions.

6. *Neither flattery nor threat is likely to succeed there.*—A compound proposition that is equivalent to the following two simple ones:

(i) *Flattery is not likely to succeed there.*
(ii) *Threat is not likely to succeed there.*

Both these propositions are of the same nature. They are categorical, negative, universal, assertory, real. Both are E propositions.

7. *None thinks the fools great but the fools themselves.*—This is a variety of the compound proposition called the exclusive proposition. It is equivalent to the following three propositions:

(i) *All who think the fools great are fools themselves.* (An A proposition.)

(ii) *No non-fools are those that think the fools great.* (An E proposition.)

(iii) *Some that are themselves fools are those that think the fools great.* (An I proposition.)

8. *Only the virtuous are happy.*—An exclusive proposition, reducible to the following three:

(i) *All really happy persons are virtuous.* (A)
(ii) *No non-virtuous persons are really happy.* (E)
(iii) *Some virtuous persons are really happy.* (I)

9. *Where there is will, there is way.*—A hypothetical proposition that, reduced to the logical form, would stand thus: *In all cases, if there is will, there is way.* It is simple, universal, affirmative, assertory, real. An A proposition.

10. *If one falls down, one does not always break one's neck.*—Logical form: *In some cases, if one is in a fallen condition, one is not in the condition of having the neck broken.* It is simple, hypothetical, negative, particular, assertory, real. An O proposition.

## Exercises

1. Classify propositions after Aristotle. Give an example of each of the sub-divisions.

2. State and explain the famous fourfold scheme of propositions.

3. State and explain with examples the division of propositions according to quantity. What is the logical character of a singular proposition?

4. Distinguish between affirmative and negative propositions. Give an example of each. What is the nature of an infinite proposition?

5. What is meant by distribution of terms? State and explain the rules of distribution.

6. Explain why A distributes its subject, E its subject and predicate, I neither subject nor predicate, and O only its predicate. Illustrate your answer.

7. Explain by means of diagrams the fourfold classification of propositions. Are these diagrams determined by the

consideration of distribution of terms in propositions?

8. What is meant by modality? What are the subdivisions of propositions according to modality? Explain them with examples.

9. How are propositions classified according to relation? Explain these classes with examples.

10. What is the relation between categorical and hypothetical propositions? Discuss whether hypothetical propositions can be negative.

11. Can hypothetical propositions be distinguished into A, E, I, and O? If so, give examples.

12. Fully explain the nature of a disjunctive proposition. How is it related to categorical and hypothetical propositions?

13. It is said that a disjunctive proposition can be reduced to hypothetical propositions. How can it be so reduced? Reduce the proposition—X is either Y or Z, to hypothetical propositions.

14. Can disjunctive propositions be distinguished into A. E, I, and O? Can they be negative?

15. Briefly but clearly state the views of modern logicians such as Russell, Johnson, Welton, Monahan, and others as to the nature and classification of propositions.

16. Explain each of the following with an appropriate example of each: Indefinite, Plurative, Indesignate, Numerically Definite, Exponible and Exceptive Propositions.

17. What is meant by quantification of the terms of a proposition? Should we quantify both the subject and the predicate of a proposition? What is Multiple Quantification?

18. Explain the terms Secundi adjacentis and Tertii adjacentis.

19. What is meant by the logical form of the proposition? How is a proposition reduced to its logical form? Reduce the following propositions to their logical form:—

(*a*) God created the world.
(*b*) Metals are heavy.
(*c*) Students are playing.
(*d*) Students play.
(*e*) What man has done man can do.
(*f*) X murdered Y.
(*g*) He runs fast.
(*h*) My going depends upon your coming.
(*i*) All men do not like to study.
(*j*) Every honest man is not trusted.
(*k*) None but a few Indians were present at the meeting.
(*l*) Men alone are present at the meeting.
(*m*) Men alone are rational.
(*n*) Most horses are beautiful.
(*o*) All except a few students are present in the class.
(*p*) All but two birds flew away.
(*q*) All men who love sports are liberal-minded.
(*r*) Sweet is the song.
(*s*) Three-fourths of the students are present in the class.
(*t*) Only a few men are geniuses.
(*u*) Few witnesses were found in the court.
(*v*) Only men have votes.
(*w*) Nearly all who were to come have come.
(*x*) Any man is not a good man.
(*y*) Few men are not poor.
(*z*) All that glitters is not gold.

20. Explain clearly the relation between the antecedent and the consequent of a hypothetical proposition.

21. Can hypothetical propositions be properly reduced to categorical ones? Reduce the following hypothetical proposition to a categorical one:—If you had not spoken the truth I should have punished you.

22. Explain why hypothetical and disjunctive propositions are called conditional.

23. Explain the logical meaning of the following words when they are used to quantify propositions:—Some, Any, All, Few, A few, Every, Many, Most Not all, No, None.

24. Express the following propositions in their simplest logical form, stating the quantity and quality of each:—

(*i*) The earth is the only planet that has an atmosphere.
(*ii*) Not to go on is to go back.
(*iii*) When beggars die there are no comets seen.
(*iv*) White cats with blue eyes are generally deaf.
(*v*) All swans are not white.

25. Discuss the character and form of conditional judgments.

26. State which of the terms are distributed in the following propositions:—

(*a*) Some of the most valuable books are seldom read.
(*b*) Every mistake is not culpable.
(*c*) He jests at scars that never felt a wound.
(*d*) No lover is he who is not always fond.
(*e*) None think the ' ›ols great but the fools themselves.
(*f*) The critical spirit is not infrequently the fault-finding spirit.

## CHAPTER IX

### IMPORT OF PROPOSITION

1. What is meant by the theory of predication? How many theories are there? Briefly explain each of them with an example.

2. Fully explain the denotative theory of predication and distinguish it from the connotative theory. Illustrate your answer. Which of these theories appears to you to be satisfactory, and why?

3. Explain with examples the predicative view of predication. Is this view satisfactory?

4. Explain with examples the indicative view of predication. Is this theory satisfactory?

5. Explain with examples the comprehensive view of predication. Discuss whether this view is satisfactory.

6. Which of the theories of predication appears to you to be the best, and why?

7. Why does Hamilton give an eight-fold scheme of classification of propositions? Explain it with examples.

8. Can propositions be expressed by equations? What is meant by an existential proposition?

# CHAPTER X

## Immediate Inference: Opposition and Eduction

### Exercises Worked Out

**Q. 1.** Examine the validity of the following inference:

*Uneasy lies the head that wears a crown; therefore, easy lies the head that wears no crown.*

**Ans.** Reduced to the logical form, the argument would stand thus:

*A head that wears a crown is a thing that lies uneasy.* (A)

∴ *A head that wears no crown is a thing that lies easy.* (A)

Invalid. The conclusion is an attempted inverse of an A proposition; but the complete inverse of A is I, and its partial inverse is O, as shown below:

*A head that wears a crown is a thing that lies uneasy.* (A)

∴ *No head that wears a crown is a thing that lies easy.* (E by obversion)

∴ *Nothing that lies easy is a head that wears a crown.* (E by conversion)

∴ *Anything that lies easy is a head that wears no crown.* (A by obversion)

∴ *Some head that wears no crown is a thing that lies easy.* (I by conversion: complete inverse)

∴ *Some head that wears no crown is not a thing that lies uneasy.* (O by obversion: partial inverse)

The given conclusion is A, and hence invalid. The valid conclusion here would be the I proposition (complete inverse) given above.

**Q. 2.** What kind of inference is the following?—

*Sweet is agreeable; therefore bitter is disagreeable.*

**Ans.** A case of Material Obversion. The conclusion here may well agree with fact, but it is formally invalid.

**Q. 3.** What conclusions, if any, can you derive from the following premise by conversion, obversion, contraposition, and inversion?—

*Anybody that holds to his own views is not an original thinker.*

**Ans.** The logical form of the proposition would be:

*Somebody that holds to his own views is a non-original thinker.*

Or,

*Some men who hold to their own views are not original thinkers.*

In any case it is an O proposition.

(*a*) An O proposition cannot be converted.

(*b*) Its obverse would be I, as—

*Somebody that holds to his own views is a non-original thinker.*

Or,

*Some men who hold to their own views are non-original thinkers.*

(c) Its contrapositive would be I, as—

*Some non-original thinker is he that holds to his own views.*

Or,

*Some non-original thinkers are men who hold to their own views.*

(*d*) Since O cannot be converted, nor can its contrapositive be contraposed, it has no inverse.

**Q. 4.** Examine the following argument:

*Absolute difference excludes all likeness; therefore, any likeness is a proof of sameness.*

**Ans.** Reduced to the logical form, the argument stands thus:

*No absolute difference is likeness.* (E)
∴ *All likeness is sameness.* (A)

Invalid. The conclusion has been arrived at by first converting the premise and then obverting the converse in this way:

*No absolute difference is likeness.* (E)
∴ *No likeness is absolute difference.* (E by conversion)
∴ *All likeness is non-absolute difference.* (A by obversion)

The valid conclusion in this case would be the A proposition obtained by obversion of the converse of the premise. In the given conclusion, which is also an A proposition, we have 'sameness' for 'non-absolute difference' as the predicate. These two terms, however, are not synonymous. So they are not interchangeable. When E obverts to A, the contradictory of the original predicate becomes the predicate of the obverse. Here the contradictory of 'absolute difference' would be 'non-absolute difference,' and not 'sameness,' which is actually its contrary.

**Q. 5.** Convert the following proposition, if possible:

*Few persons can keep a secret.*

**Ans.** Its logical form would be:

*Some persons are not capable of keeping a secret.* (O)

An O proposition cannot be converted.

**Q. 6.** Draw as many conclusions as you can from the following premise:

*Every event has a cause.*

**Ans.** Its logical form would be:
*All events are caused.* (A)

(*a*) Converse:
*Some things caused are events.* (I)

(*b*) Obverse:
*No events are not-caused.* (E)

(*c*) Obverted converse:
*Some things caused are not non-events.* (O)

(*d*) Contrapositive:
*No non-caused things are events.* (E)

(*e*) Obverted contrapositive:
*All non-caused things are non-events.* (A)

(*f*) Partial Inverse:
*Some non-events are not caused.* (O)

(*g*) Complete Inverse:
*Some non-events are not-caused.* (I)

(*h*) Subalternate:
*Some events are caused.* (I)

(*i*) Modal consequence:
*All events may be caused.* (A)

(*j*) Change of Relation:
*In all cases, if there is an event, it is caused.* (A)

**Q. 7.** Obvert the premise:
*Graduates alone are eligible for the post.*

**Ans.** Logically reduced, the proposition would be:
(i) *All who are eligible for the post are graduates.* (A)
(ii) *No non-graduates are eligible for the post.* (E)

The obverse of (*i*) would be:

*None eligible for the post are non-graduates.* (E)

The obverse of (*ii*) would be:

*All non-graduates are not-eligible for the post.* (A)

**Q. 8.** Examine the argument:

*Books are a source of instruction; therefore our knowledge must come from books.*

**Ans.** Logically the argument stands thus:

*All books are a source of instruction.* (A)

∴ *All sources of instruction are books.* (A)

Invalid. By converting the given premise we get the conclusion:

*Some sources of instruction are books.* (I)

The given argument is a case of simple conversion. But simple conversion is possible only where the predicate is co-extensive with the subject. Here it is not so.

**Q. 9.** Examine the validity of the argument:

*Revolutionaries are reformers; therefore reformers are revolutionaries.*

**Ans.** Since the whole argument consists of two indesignate propositions, one can't be sure of their intended logical form.

If they were intended to be universal, the argument would stand thus:

*All revolutionaries are reformers.* (A)

∴ *All reformers are revolutionaries.* (A)

Invalid. A has been simply converted to A, although the predicate ('reformers') is undistributed.

But the argument may be reduced to the following form:

*Some revolutionaries are reformers.* (I)

∴ *Some reformers are revolutionaries.* (I)

Valid. I converts to I.

**Q. 10.** Examine the following:

*The pen is mightier than the sword; therefore the sword avails less than the pen.*

**Ans.** The logical form of the argument would be:
*The pen is mightier than the sword.* (A)
∴ *The sword is less availing than the pen.* (A)

But this does not fit in with any of the recognised forms of inference. Unless the whole argument is shorn of its figurative character, it is scarcely above nonsense. Nevertheless, it is an attempt at inference by converse relation, and as such not altogether mistaken, if we grant the premise.

## Exercises

1. Show that opposition is a relation between propositions and is also a doctrine of immediate inference.

2. What are the different kinds of opposition existing between A, E, I, and O? Show them by the square of opposition.

3. Define opposition and clearly explain its nature. Illustrate your answer.

4. Define contrary opposition, sub-contrary opposition, contradictory opposition, and subalternation. Give a concrete example of each.

5. State with reasons the rules of inference by opposition. Are subalternation and subcontrariety oppositions proper? Illustrate your answer.

6. What inferences by opposition can you draw from the truth and falsity of the following propositions:—

(*a*) Many men are unhappy.
(*b*) Every bird is not musical.
(*c*) All men are rational.
(*d*) No elephant is without a trunk.
(*e*) No men are perfect.
(*f*) Some peoples are not free.

7. What are the different kinds of inference? Distinguish between immediate inference and mediate inference and also

between formal and material inference. Illustrate your answer.

8. Define inference, and explain its nature clearly. How is it related to judgments and terms? Is immediate inference inference proper?

9. Explain with examples the opposition of singular propositions.

10. Reduce the hypothetical proposition, If S is M it is P, to the four traditional forms, and show how the doctrine of opposition applies to them.

11. Reduce the disjunctive proposition, S is either P or Q, to the four traditional forms, and show how the doctrine of opposition applies to them.

12. What is the logical nature of contradiction? Can there be pure denial?

13. Define eduction. What are its different forms? Illustrate your answer.

14. Define conversion and state the rules of inference required in conversion. Why does A convert per accidens, E and I simply, and O not at all? Can O be converted in any case and can A ever be converted simply? Illustrate your answer.

15. Define obversion and state the rules of inference required in obversion. Obvert A, E, I and O. What is material obversion? Is it justified?

16. Show that conversion rests upon the principle of identity and obversion upon the principles of contradiction and of excluded middle. Show that obvertend and obverse are equivalent propositions.

17. What is obverted conversion? Explain its nature with examples.

18. Define contraposition. Is it an independent mode of inference? Contrapose A, E, I and O. Distinguish between partial contrapositive and full contrapositive.

19. How is it that A and O can be contraposed simply and E by limitation? Why cannot I be contraposed? What objections are advanced against obversion and contraposition as modes of inference?

20. Define inversion. Distinguish between partial inversion and full inversion. Invert A, E, I and O. What objections may be advanced against inversion as a mode of inference? How would you answer these objections?

21. Define the following and explain their nature with illustrations:—Inference by Added Determinants, Immediate Inference by Complex Conception, Immediate Inference by Converse Relation, Immediate Inference by Change of Relation, Immediate Inference by Modal Consequence.

22. What is meant by material forms of immediate inference? Why are they called material? What are they? Give an example of each.

23. Explain the terms convertend, converse, obverse, obvertend, contraponend, contrapositive, invertend and inverse.

24. Obvert, convert, contrapose and invert the following propositions:—

(*a*) All that glitters is not gold,
(*b*) None but the brave deserves the fair,
(*c*) Water is liquid,
(*d*) All birds lay eggs,
(*e*) No honest man deceives,
(*f*) All men are not happy,
(*g*) Some flowers are yellow,
(*h*) Some men are not honest,
(*i*) If a man is virtuous he may be trusted,
(*j*) If you come I need not be pleased,
(*k*) If any man lies he is never trusted,
(*l*) If a man is honest he need not always be successful.
(*m*) Flowers are either white or not-white,
(*n*) God is either all-powerful or is limited in power.

25. Distinguish between obversion, contraposition and inversion, giving concrete examples of each. What kind of inference have we in the following: "Only the ignorant despise knowledge, therefore some persons despising knowledge are ignorant"? Supposing the proposition, Some stones are not minerals, to be false, what can you know about the truth and falsity of its opposites?

26. (*a*) Prove by means of contradictory propositions that sub-contrary propositions cannot both be false.

(*b*) Show by means of the sub-contrary propositions that contrary propositions may both be false.

27. (*a*) Is it ever possible to derive a conclusion from a single premise? If it is, name and define the different ways of doing it.

(*b*) Show what conclusions can be derived from the following as premise—All men are not poets.

28. Give the converse, obverse and contradictory, where possible, of the following propositions:—

(*a*) The earth is the only planet that has an atmosphere.
(*b*) Not to go on is to go back,
(*c*) When beggars die there are no comets seen,
(*d*) White cats with blue eyes are generally deaf,
(*e*) All swans are not white.

29. Give the quality and quantity of each of the following propositions, reduce each to its logical form, and give the obverse, contrapositive and inverse, if possible, of each:—None think the fools great but the fools themselves, Things are not what they seem, One of you at least should be able to answer this question.

30. Answer any two of the following three questions:—

(*a*) Examine the following immediate inference:·

Absolute difference excludes all likeness; any likeness is a proof of sameness.

(*b*) Explain the relation of immediate inference to the laws of thought.

(*c*) What do you understand by the opposition of propositions? What propositions can be inferred as true, false or doubtful if the following proposition is false—'Some happy men are discontented'?

31. What is Immediate Inference? "All students of logic are not strong in logic"; apply the various processes of Immediate Inference to this proposition, indicating the result in each case.

32. Draw the inferences which follow from the proposition, None but the industrious deserves success.

33. Define and explain with illustrations conversion by negation. Is it a form of mediate or immediate inference?

34. Explain conversion and contraposition, and give the inferences that follow by those processes from each of the following: There is no man that is not naturally good: Men are never happy if they are not virtuous.

35. (*a*) All lawyers are not knaves.

(*b*) None but graduates are eligible. Put each of the above propositions into exact logical form, and consider what propositions can be inferred by conversion and by obversion.

36. Are the following inferences correct?

(*a*) If all good people are happy, unhappiness is an indication of vice.

(*b*) Warmth is agreeable; therefore cold is disagreeable.

(*c*) No Intermediate student reads Mill's Logic; therefore there are some other students who read the book.

37. Convert—None but elements are metals.

Obvert—No men are perfect.

Contrapose—All men are mortal.

What propositions are true, false, or doubtful,

(*a*) when A is true,

(*b*) when O is false?

38. Prove by means of the rule of sub-contrary opposition that two contrary propositions cannot both be true.

39. State the quality and the quantity of the following propositions, and give the obverse and the contrapositive of them all:—

(*a*) Only the educated are fit to vote.

(*b*) A few drops of rain are not of much consequence.

(*c*) Man proposes, God disposes.

(*d*) A few men will not suffice to remove this big table.

40. Test the validity of the following immediate inferences:

(*a*) Prudence is a virtue.
∴ A prudent man is a virtuous man.

(*b*) Virtue leads to happiness.
∴ Happiness leads to virtue.

(*c*) A professor is a man.
∴ A bad professor is a bad man.

(*d*) Truth is beauty.
∴ Beauty is truth.

(*e*) Only children behave in this way.
∴ Everyone who behaves thus is a child.

(*f*) No one is admitted without payment.
∴ All who are admitted are persons who paid.

(*g*) Uneducated men make wrong decisions.
∴ No right decisions are taken by the educated.

(*h*) Only fools rush in where angels fear to tread.
∴ All cautious men are wise.

(*i*) Only the ignorant despise knowledge.
∴ Some persons who despise knowledge are ignorant.

(*j*) Most shopkeepers are honest men.
∴ Some dishonest men are not shopkeepers.
(*k*) All mangoes are not sweet.
∴ All sweet things are not mangoes.
(*l*) All kings are men.
∴ A majority of kings is a majority of men.
(*m*) No dogs are hogs.
∴ Some creatures that are not dogs are hogs.
(*n*) Anybody is not a good writer.
∴ Somebody is a good writer.

# CHAPTERS XI-XII

## Pure Syllogisms and Diagrammatic Representation of Syllogisms

### Exercises Worked Out

*Arguments Tested*

1. Man created sin.
God created man.
∴ God created sin.

Logically put, the argument stands thus:
Man is the creator of sin.
God is the creator of man.
∴ God is the creator of sin.

There is no middle term here, the terms being 'man,' 'the creator of sin,' 'God,' and 'the creator of man.' It involves *the fallacy of four terms*.

2. All criminal actions ought to be punished.
Prosecutions for theft are criminal actions.
∴ Prosecutions for theft ought to be punished.

There are apparently three terms here—'criminal actions,' 'to be punished,' and 'prosecutions for theft.' But the middle term 'criminal actions' bears two different meanings; in the major premise it means 'crimes,' and in the minor premise 'cases relating to crimes.' Hence the argument involves *the fallacy of ambiguous middle,* which is only a variety of the fallacy of four terms.

3. Solon was really competent to rule, for we know he was wise, and it is the wise who are fitted to rule.

Logically stated, the argument stands more or less thus:

All persons competent to rule are wise.
Solon is wise.
∴ Solon is a person competent to rule.

Invalid. The middle term, *wise,* has not been distributed in any of the premises. Hence *the fallacy of undistributed middle.*

4. He that is of God heareth God's words. Ye, therefore, hear them not, for ye are not of God.

Logically stated, the argument stands somewhat thus:

All men of God are hearers of God's words.
You are not men of God.
∴ You are not hearers of God's words.

Invalid. *Illicit major.* The major term, *hearers of God's words,* is undistributed in the major premise, it being an A proposition. But it has been distributed in the conclusion, which is a negative proposition.

5. Mercy but murders, pardoning those that kill.

Logically stated, the argument stands somewhat thus:

Pardoning those that kill is to murder.
To show mercy to those that kill is pardoning those that kill.
∴ To show mercy to those that kill is to murder.

Formally valid being in Barbara. (But see *poste.*)

6. You are not what I am; but I am a man; therefore you cannot be a man.

Logically put, the argument stands thus:

I am a man.
You are not I.
∴ You are not a man.

Invalid. *Illicit major.* The major term, *man,* though undistributed in the major premise, has been distributed in the conclusion.

7. An ounce of carbon will cost you dear; for don't you know that diamond sells at a prohibitive price, although it's only carbon?

Logically put, the argument stands somewhat thus:

All diamonds are dear.
All diamonds are carbon.
∴ All carbon is dear.

Invalid: *Illicit minor.* The minor term, *carbon,* in the minor premise is undistributed, but it has been distributed in the conclusion.

8. The news is too good to be true.

Logically the argument is something like the following:

No too good news is true.
This is too good news.
∴ This is not true.

Formally valid, being in Celarent.

9. He must be a Scotsman, for no Scotsman can see the force of a joke.

Logically, it would be more or less as follows:

All Scotsmen are incapable of seeing the force of a joke.
He is incapable of seeing the force of a joke.
∴ He is a Scotsman.

Invalid. *Undistributed Middle.* The middle term, *in-*

*capable of seeing the force of a joke,* is undistributed in both the premises, being, in both the cases, predicate of affirmative propositions.

**N.B.** It is sometimes possible to put an argument into the logical form in different ways, as shown below:

No Scotsman is able to see the force of a joke.
He is not able to see the force of a joke.
∴ He is a Scotsman.

Invalid. It involves *the fallacy of negative premises.* From two negative premises no conclusion follows—not even a negative conclusion.

10. Learned men sometimes become mad. As he is not learned, there is no danger to his sanity.

Logically the argument may be put thus:

Some learned men are likely to become mad.
He is not a learned man.
∴ He is not likely to become mad.

Invalid. *Illicit major.* The major term, *likely to become mad,* is undistributed in the major premise, but distributed in the conclusion.

**Various Syllogistic Rules Proved**

1. *When the conclusion is universal, the middle term is distributed only once.*

A universal conclusion can be either A or E.

(*a*) If it is A, both the premises must be A. An A proposition distributes only one term, *viz.,* the subject. Thus between the two A propositions, only two terms are distributed. One of these is again distributed in the conclusion, and this is the minor term. This being so, the minor term requires to be distributed in the minor premise. So the question of the middle term's being distributed in the minor premise does not arise. Thus only the major premise is left for its being distributed, and as the major premise, like the

minor, distributes only one term, *viz.*, the subject, the middle term can be distributed in the whole syllogism only once.

(*b*) If the conclusion is E, one of the premises must also be E, and the other A. An E proposition distributes both the subject and the predicate, while A distributes only the subject. Thus altogether three terms are distributed between the premises. The conclusion, being E, must distribute both the terms, and these two therefore require to be distributed in the premises. This being so, only one opportunity is left in the premises for the middle term to be distributed.

2. *When the middle term is distributed twice, the conclusion is particular.*

No term can occur twice in the same proposition without tautology, and even when it does so occur, it is, no doubt, the same word, but certainly not the same term, as in one position it stands as the subject-term and in the other as the predicate-term. So if the middle term be distributed twice in a syllogism, it would be distributed in both the premises, and not in any one of them. If it be distributed in two affirmative premises, they would be A propositions. Since an A proposition can distribute only the subject-term, the middle term, in such a case, must be the subject of both the premises. Thus none of the premises would distribute either the major term or the minor term. So the conclusion can be only an I proposition, *i.e.*, particular.

If, however, the middle term be distributed twice when one of the premises is negative, only one of the other two terms would have an opportunity of being distributed. But one of the premises being negative, the conclusion also must be negative. A negative conclusion would distribute its predicate, which, however, must be the major term of the syllogism. For the major term's to be distributed in the conclusion, it must be distributed in the major premise. The minor term can thus have no opportunity of being distributed in the minor premise, and so it can have no opportunity

either of being distributed in the conclusion, which would thus be an O proposition, *i.e.*, particular.

3. *Only in the first figure the conclusion can be A.*

When the conclusion is A, both the premises are A as well. As the minor term, which is the subject of the conclusion, is distributed in this case, it is distributed in the minor premise also. Since an A proposition distributes its subject only, and not the predicate, the minor term, in order to be distributed in the minor premise, must be its subject. So the middle term is left undistributed in the minor premise, where it forms the predicate. But the middle term must be distributed in the premises at least once. So it must be the subject of the major premise. Thus when the conclusion is A, the middle term is the subject of the major premise, and predicate of the minor. The resulting syllogism is therefore in the first figure.

4. *When the minor premise is negative, the middle term is distributed only once.*

When the minor premise is negative, the major is A, and the conclusion negative. The negative conclusion distributes the major term, which, therefore, is also distributed in the major premise. But the major premise, being A, distributes only the subject, which, in this case, is the major term, and not the middle term. Hence only the minor premise is left for the middle term's being distributed, that is to say, only one opportunity is left for its distribution.

5. *When the minor term is the predicate of the minor premise, the conclusion cannot be A.*

The minor term as predicate in the minor premise can be distributed only when it is a negative proposition. In that case, the conclusion also is negative. A negative conclusion cannot be an A proposition.

If, however, the minor term is undistributed in the minor premise, it must be undistributed in the conclusion as well.

But the minor term is always the subject of the conclusion. The subject being undistributed, the conclusion must be particular, and not an A proposition.

6. *When the minor premise is negative, the major premise is universal.*

If the minor premise be negative, the major premise must be affirmative, otherwise no conclusion would follow. And in this case, the conclusion must be negative. A negative conclusion must distribute the major term; so it must also be distributed in the major premise. As the major premise is affirmative, it cannot distribute its predicate. So the major term can be distributed only as the subject of the major premise, which thus becomes an A proposition, *i.e.*, an universal (affirmative) proposition.

7. *An O proposition can never be a premise in the first figure.*

In the first figure, the middle term is subject in the major premise, and predicate in the minor.

An O proposition does not distribute its subject. So if the major premise be O, the middle term would remain undistributed. When the major premise is O, the minor premise is A, and has the middle term for its predicate. But A does not distribute its predicate. Thus in the combination OA, the middle term remains undistributed, and no conclusion follows.

If O is the minor premise, the major premise is A, and the conclusion O. O distributes the predicate, which, in this case, is the major term. So for a valid conclusion, the major term must be distributed in the major premise, which, being A, can distribute the subject only. But this is not possible in the first figure, which must have the middle term for subject in the major premise. So the combination AO in the first figure yields no conclusion.

Thus O cannot be a premise in the first figure.

8. *An O proposition can never be a premise in the fourth figure.*

In the fourth figure, the middle term is predicate in the major premise, and subject in the minor.

If O happens to be the major premise, the major term remains undistributed, and cannot therefore be distributed in the conclusion. But with O as a premise, the conclusion must be O, which would distribute the predicate, and the predicate would be the major term. That would be unwarranted.

If, however, O be the minor premise, we must have A for the major premise. The middle term would remain undistributed in both; for neither the subject of O, nor the predicate of A, is distributed. Thus no conclusion would follow.

9. *An O proposition can be major premise only in the third figure.*

(*a*) In the first figure, the middle term is subject in the major premise, and predicate in the minor.

With O as major premise, we must have A for the minor. O does not distribute its subject, nor A, its predicate. So OA in the first figure does not distribute the middle term even once. Hence no conclusion follows.

(*b*) In the second figure, the middle term is predicate in both the premises.

If O be the major premise, the major term, which, in this case, is subject, remains undistributed, and yet it is distributed in the conclusion, which must also be O. But as that would lead to illicit major, no conclusion would follow.

(*c*) In the fourth figure, the middle term is predicate in the major premise, and subject in the minor.

If O is made the major premise, the major term, as its subject, remains undistributed, and yet is distributed in the conclusion, which is O. This also leads to illicit major.

(*d*) In the third figure, the middle term is subject in both the premises.

If the major premise is O, it distributes its predicate, *i.e.*, the major term. The conclusion would also be O, but it would then legitimately distribute the major term as its predicate. Again, with O for the major premise, we have A for the minor. And since A distributes the subject, the middle term would be distributed there. A does not distribute the predicate, which, in this case, would be the minor term. But then the conclusion, being O, would not distribute its subject, which would be the minor term. So no question would arise on that score. Hence O can be major premise in the third figure, and the third figure alone.

10. *An O proposition can be minor premise in the second figure alone.*

(*a*) [For 1st figure, see 7 above.]

(*b*) In the third figure, the middle term is subject in both the premises.

If O be the minor premise, the major premise must be A, and the conclusion, O. The conclusion would then distribute the predicate, *i.e.*, the major term, without its being distributed in the major premise, which, as A, would not distribute its predicate. Hence with O for minor premise in the third figure, there is illicit major.

(*c*) In the fourth figure, the middle term is predicate in the major premise, and subject in the minor.

Since O is the minor premise, the major premise is A. The middle term remains undistributed in the major premise, as it is predicate there. In the minor premise, too, where it is subject, the middle term is not distributed. So with O as minor premise in the fourth figure, no conclusion follows.

(*d*) In the second figure, the middle term is predicate in both the premises.

The minor premise being O, the major premise is A, and the conclusion, O. The major term, which is the subject of the A proposition, is distributed. So its distribution in the conclusion is valid. The minor premise, being O, distributes

the middle term. The minor term remains undistributed in the conclusion as in the minor premise, where it is the subject. So the combination AO in the second figure yields a valid conclusion. Hence in the second figure, O can stand as minor premise.

### Exercises

1. Define syllogism and clearly explain its nature and its importance in formal logic. Why is it regarded as a demonstrative form of inference?

2. How many terms and propositions does a syllogism contain? Define major term, major premise, minor term, minor premise, middle term and conclusion. What is the function of the middle term?

3. Is the order of propositions in a syllogism improtant? Why is it said that the conclusion of a proposition follows necessarily from the premises?

4. What is an antilogism? Explain it with an example and contrast it with a syllogism.

5. Name and explain the main axioms of pure syllogism.

6. Clearly explain the dictum of Aristotle and point out what consequences follow from it. Why does Aristotle regard it as the formal ground of pure syllogism?

7. Deduce the fundamental rules of syllogism from the dictum of Aristotle.

8. Clearly explain the dictum Nota notae. Why does Mill prefer this dictum to that of Aristotle as the ground of pure syllogism?

9. Show that affirmative categorical syllogisms rest on the principle of identity and negative categorical syllogisms on the principle of contradiction, and that pure hypothetical syllogisms require in addition the principle of sufficient reason.

10. Explain why every syllogism should have three and only three terms, and also should have three and only three propositions. Illustrate the fallacies which arise from the violation of these rules.

11. Explain why the middle term of a syllogism should be distributed at least once. Name and illustrate the fallacy which arises from the violation of this rule. Illustrate this rule by Euler's diagrams.

12. State and explain why no term may be distributed in the conclusion which has not been distributed in one of the premises. Name and illustrate the fallacies which arise from the violation of this rule.

13. Explain why from two negative premises no conclusion can be drawn. Name and illustrate the fallacy which arises from the violation of this rule. Can a conclusion ever be drawn from two negative premises? Illustrate the rule by Euler's diagrams.

14. Explain why if one of the premises is negative, the conclusion must be negative, and if both the premises are affirmative, the conclusion must be affirmative.

15. Prove that if the conclusion is negative, one of the premises must be negative, and if the conclusion is affirmative, both the premises must be affirmative.

16. Prove that from two particular premises no conclusion can be inferred. Why is this rule regarded as a corollary of the fundamental rules of syllogism? Does this rule admit of any exception?

17. Prove that if one premise be particular, the conclusion must be particular. Why is this rule regarded as a corollary of the fundamental rules of syllogism?

18. Prove that if the major premise be particular and the minor negative, nothing can be inferred. Why is this rule regarded as a corollary of the fundamental rules of syllogism?

19. What are the fundamental rules of syllogism and why are they regarded as fundamental?

20. Is syllogistic reasoning nothing but substitution.

21. Define figure and state its different forms.

22. Define mood. How many possible moods are there? How many moods are valid? How many moods are fundamental, and what are they?

23. State and prove the special rules of the first figure and determine the valid moods of the figure.

24. State and prove the special rules of the second figure and determine the valid moods of the figure.

25. State and prove the special rules of the third figure and determine the valid moods of the figure.

26. State and prove the special rules of the fourth figure and determine the valid moods of the figure.

27. Show clearly that the first figure and the special rules of that figure rest upon the dictum of Aristotle.

28. In how many valid moods is the middle term distributed twice? Illustrate your answer.

29. Explain weakened syllogism, subaltern mood, and strengthened syllogism. How many weakened syllogisms are there? Name them and show why they are weakened. How many strengthened syllogisms are there? Name them and show why they are strengthened.

30. How many weakened syllogisms are at the same time strengthened? Is there any weakened syllogism which is not strengthened? If so, what is it and why is it so? What is the total number of strengthened and weakened syllogisms? If we omit them from the list of valid moods, how many valid moods remain? Name them.

31. How many syllogisms can prove A, how many can prove E, how many can prove I, and how many can prove O? Name what moods can prove E, what moods O, what moods I, and what moods A.

32. Why is the first figure regarded as most natural? What are its merits? What are the uses of the second, third, and fourh figures? Is the fourth figure useless?

33. What is a pure hypothetical syllogism, and what is a pure disjunctive syllogism? Give an example of each.

34. What is reduction? Is the doctrine of reduction indispensable in logic? What is the utility of reduction?

35. Distinguish between direct and indirect reduction. What moods did Aristotle reduce indirectly and why? Reduce Baroco and Bocardo both directly and indirectly.

36. Reduce Camenes, Felapton, Bramantip, and Dimaris directly, and two of them indirectly as well.

37. Can pure hypothetical syllogisms be reduced? If so, reduce one such syllogism directly.

38. Why does the second figure prove only negative propositions, and the third figure only particular propositions? Illustrate your answer.

39. State the following arguments in figure and mood, and determine whether they are valid or not:—

(a) It is impossible that thought can be a function of matter, because all the functions of matter are modes of motion, which thought is not.

(b) Beggars who have no property can receive no injustice, because injustice is nothing but violation of property.

(c) The formal study of logic is useless, for many persons who have never studied logic can reason accurately.

(d) Some of Galileo's contemporaries argued, There can be no truth in Galileo's assertion that Jupiter has moons, because they are invisible to men and can therefore have no bearing on the interests of mankind, and there can be nothing in this world that has not some relation to man.

40. (a) In which of the following syllogisms is the conclusion valid under any mood and figure, and in which is it invalid, and why?—EAE, EAA, EIO, IEI.

(b) Prove that if the conclusion be universal, the middle term must be distributed only once in the premises.

41. State any four of the following arguments in the syllogistic form, simplifying the premises and arranging them according to mood and figure, and show whether or not the reasoning is correct, pointing out fallacies if any:

(a) Animals cannot live without atmosphere. There is no atmosphere on the moon. Therefore there can be no animal life on the moon.

(b) The sky being cloudy, the day is sultry.

(c) Where there is social progress there must be liberty of individuals, and where there is liberty of individuals there must be inequality of wealth and standing. Therefore inequality and progress are inseparable.

(d) None but material bodies gravitate; air gravitates; therefore it must be material.

(e) Solon was really competent to rule, for we know he was wise, and it is the wise only who are fitted to rule.

42. Prove that O cannot be a premise in the first figure, nor a minor premise anywhere but in the second.

43. Give a concrete example of Disamis and Bramantip, and reduce each of them by both the direct and the indirect methods.

44. Discuss the importance of Aristotle's dictum de omni et nullo. What is Reduction? Is it necessary? Give a concrete example of Disamis and reduce it both by the direct and indirect methods.

45. (a) Prove that O cannot be the major premise in the second figure or the minor in the third. (b) If the major term of a syllogism be the predicate of the major premise, what do you know about the minor premise? (c) Test AEE in each figure. (d) If the premises of a syllogism are false, does this make the reasoning false? Illustrate your answer with a concrete example.

46. (a) Prove that in the second figure the major premise must be universal. (b) Given that the major premise of a valid syllogism is affirmative and that the major term is distributed in the premise and the conclusion, while the minor term is undistributed in both, determine the syllogism.

47. Indicate the different modes of testing the validity of a syllogistic argument, illustrating your answer with examples.

48. Determine the mood and the figure in which the major term is distributed in the major premise but is undistributed in the conclusion. Find out the moods which are valid in all the figures.

49. Two propositions are given having a common term. (a) If they are I and A, show that they justify either no conclusion or two conclusions. (b) If they are I and E, show that they always lead to a single conclusion.

50. Prove any two of the following:—(a) If the conclusion be A the argument must be in the first figure; (b) if the conclusion be universal the middle term can be distributed but once; (c) if both the premises be particular no conclusion can be drawn. Is there any exception to (c)?

51. (a) Show that O cannot be a premise in the first or fourth figure. (b) Why is it said that A is the most difficult conclusion to establish by syllogism and the most easy to overthrow? (c) Test EAE in each figure.

52. Prove that the conclusion is particular when a premise is particular. Is the converse of this rule true?

53. Prove that (a) if the middle term be distributed in both the premises the conclusion cannot be universal; (b) the major premise must be universal in the first and second figure; (c) if the minor premise be negative the major must be universal.

54. Construct syllogisms to prove the following conclusions, stating the figure and the mood of each: (a) John is bound to succeed in life. (b) That train does not stop at this station.

55. Examine the conclusions of AOE, OAE, OIE, and show whether they are valid under any of the syllogistic rules, giving your reasons.

56. Show that if the middle term of a valid syllogism is distributed in both the premises the conclusion must be particular.

57. Illustrate the valid moods by Euler's diagrams.

## CHAPTER XIII

### Mixed Syllogisms

1. Define mixed syllogism. Why is it called mixed? What are its forms? Illustrate them.

2. Distinguish between pure and mixed syllogisms, indicating their forms and tests.

3. State the rules of inference of mixed hypothetical syllogism and point out the fallacies which arise from their violation. Illustrate your answer. Draw as many conclusions as you can from the proposition, If S is M, it is P.

4. Name and explain with examples the different moods of mixed hypothetical syllogism. What are the sub-forms of these moods? Illustrate them.

5. Define mixed disjunctive syllogism and clearly explain its nature, with illustrations.

6. Name and exemplify the two moods of the mixed disjunctive syllogism.

7. When can we draw two conclusions from a disjunctive major premise, and when can we draw four conclusions from it? Illustrate your answer.

8. State the rules of disjunctive syllogism and illustrate the fallacies which arise from their violation.

9. (a) Are mixed hypothetical and disjunctive syllogisms syllogisms proper? Fully discuss the question. (b) Draw as many conclusions as you can from the propositions, This flower is either green or white, and This flower is either green or not-green.

10. What is dilemma? Clearly explain its nature, and name and illustrate its different forms.

11. Is it true to say that a dilemmatic argument is more often fallacious than not? Fully discuss this question.

12. What are the different ways in which a dilemma can be refuted? Illustrate your answer. What are the horns of a dilemma?

13. How is a dilemma rebutted? Does every dilemma admit of rebutting? Illustrate your answer. What conditions shoud a dilemma fulfil in order to be valid?

14. If A is true, B is true.
If C is true, B is not true.

Show what conclusion follows from these premises:
(*a*) If A is true, and
(*b*) If C is true.

15. Rebut the following dilemmas:

(*a*) If emigrants are useless, they are a burden to the colonies; If they are useful, they are a loss to the mother country. But they are either useless or useful.

∴ Emigration is either a burden to the colonies, or a loss to the mother country.

(*b*) If there is censorship of the press, abuses that should be exposed will be hushed up;

If there is no censorship, truth will be sacrificed to sensation.

But there must either be censorship, or not.

∴ Either abuses that should be exposed will be hushed up, or truth will be sacrificed to sensation.

(*c*) If a pupil is fond of learning, he needs no stimulus; if he dislikes it, no stimulus would be of any avail. He must either be fond of learning, or he must dislike it. Therefore stimulus is either needless or of no avail.

16. Construct a dilemma to show that money is useless.

17. Little Jit was called a fool to his face by his chum for declaring that a certain picture book was very instructive, whereupon he urged upon his friend somewhat as follows:

If I am a fool, my opinion that the book is instructive is right, for I have derived much benefit from it; and if I

am as intelligent as you are, my opinion is quite right and yours is absolutely wrong, for I can judge about it all right. I am either a fool as you call me, or I am as intelligent as you are. In any case, my opinion cannot be gainsaid.

What do you think of this argument?

18. (*a*) If a student is diligent, he passes the examination. He passes the examination. Therefore he is diligent. (*b*) If a person is guilty, he will be punished. But he is not guilty. Therefore he will not be punished. Reduce each of the above hypothetical syllogisms to the categorical form, and then test its validity as a categorical syllogism.

19. Put the following argument into dilemmatic form, and rebut it by framing a counter-dilemma :—An escort is unnecessary ; for if you are well received, it will be needless ; and if you are not well received, it will raise suspicion.

20. Examine the following dilemma :—If logic deals with the matter of thought, it must either consider the whole of it, and then be identical with all science, or consider only a part of it without being able to give a reason why it should choose one part rather than another.

## CHAPTER XIV

### The Enthymeme, Sorites, and Epicheirema

1. Define Enthymeme. Exemplify enthymeme of the first order, of the second order, and of the third order. Why do men resort to enthymematic arguments?

2. Define 'train of reasoning'. Distinguish clearly, with examples, between progressive and regressive trains of reasoning. What is a prosyllogism, and what is an episyllogism? Illustrate your answer.

3. Define Sorites. What are its different forms? Illustrate Aristotelian and Goclenian sorites, and compare them.

4. State and explain the rules of inference of Goclenian and Aristotelian sorites, and illustrate the fallacies which arise from their violation.

5. Should sorites be only in the first figure? Can there be any sorites in the second or in the third figure? Illustrate your answer.

6. Define Epicheirema and illustrate its different forms.

7. Distinguish between Sorites and Epicheirema.

8. Give a sorites of the Aristotelian form (symbolical), and break it up into its constituent simple syllogisms.

## CHAPTER XV

### FUNCTION, VALIDITY, AND RANGE OF SYLLOGISMS

1. What is the value of syllogism? Is it useless? Is Mill's criticism of the value of syllogism sound?

2. Is syllogistic reasoning fallacious? Examine Mill's view regarding the validity of syllogism.

3. Is Mill justified in holding that all reasoning is from the particular to the particular and there is no need to argue from the general to the particular?

4. Is syllogism the only mode of reasoning, as some logicians hold? Discuss its importance in logic.

5. Explain clearly the range and limits of syllogism. Is it the only form of deductive reasoning? Discuss the question.

6. What are the different forms of synthesis according to Bradley? Explain them with illustrations.

7. Why does Russell hold that syllogism is not the only type of mediate inference? State and explain his classification of relations from which inferences may be drawn.

8. Point out the relation between deduction and induction

9. Does the syllogism really extend our knowledge? Fully discuss this question.

10. Explain and examine the objection of Mill that the syllogism as a mode of argument which involves the fallacy of petitio principii.

# CHAPTER XVI

## Formal Fallacies

### Exercises Worked out.

1. *The end of life is its perfection; death is the end of life; therefore death is the perfection of life.*

The conclusion is quite obvious—'Death is the perfection of life.' The subject of the conclusion is always the minor term of a syllogism. So 'Death' is the minor term here. The premise in which it occurs is the minor premise. So the minor premise here is: 'Death is the end of life.' The predicate of the conclusion is always the major term, which, in this case, is: 'the perfection of life'. The premise in which it occurs is the major premise. So the major premise here is: 'The end of life is its perfection'. Now we can thus put the syllogism into the logical form:

The end of life is the perfection of life.
Death is the end of life.
∴ Death is the perfection of life.

The middle term here is: 'The end of life'. The argument is in Barbara, and apparently all right. But the middle term, 'the end of life', in the major premise means 'the ideal of life', while, in the minor premise, it means quite a different thing, *viz.*, 'the termination of our earthly existence'. Hence the syllogism involves *the fallacy of ambiguous middle,* which is a form of the fallacy of equivocation, which, in its turn, is actually a fallacy of four terms, as every ambiguous term in a syllogism is logically equivalent to two terms.

2. *This substance cannot be gold, as it is not malleable.*

This is actually an enthymeme, as it contains only two propositions instead of three.

The conclusion obviously is: 'This substance cannot be gold.' The other proposition, 'it is not malleable', is the minor premise, for it contains the minor term, 'it', which actually refers to the subject of the conclusion, *viz.*, 'this substance'. The major premise is not given.

But the major premise must contain the major term and the middle term. The predicate of the conclusion is the major term. So we have it there. It is 'gold'. The middle term is 'malleable', as it is the only other term, besides the minor term, to occur in the minor premise. Now we can perhaps construct the major premise.

But any proposition containing these two terms, the major and the middle, would not do. We must again refer to the minor premise and the conclusion.

The conclusion, logically put, would be: 'This substance is not gold'. It is an E proposition. The minor premise, logically put, would be: 'This substance is not malleable'. It is also an E proposition. So we must have a universal affirmative proposition (A) for the major premise. Thus the major premise comes to be: 'Gold is malleable'. So at last we have the syllogism :

Gold is malleable.
This substance is not malleable.
∴ This substance is not gold.

Valid, being in Camestres.

3. *Queen Caroline had so much influence over King George II as virtually to rule him. So people said it was she who actually ruled England and Hanover, and not the King.*

What is the conclusion in this argument? Obviously, 'Queen Caroline is the person to rule England and Hanover'. Now it is easy to find out the minor premise, which is: 'Queen Caroline is the person to rule King George II.' The major premise then is: 'King George II is the person to rule England and Hanover.' So the syllogism stands thus:

King George II is the person to rule England and Hanover.
Queen Caroline is the person to rule King George II.
∴ Queen Caroline is the person to rule England and Hanover.

Invalid, for there is no middle term here. The terms are: (1) King George II. (2) The person to rule England and Hanover, (3) Queen Caroline, and (4) The person to rule King George II. The syllogism thus involves *the fallacy of four terms.*

4. *Only fools behave thus, but I behave quite otherwise.*

An enthymeme that suggests the conclusion by suppressing it. The conclusion obviously is: 'I am not a fool.' Logically the argument stands thus:

All who behave thus are fools.
I am not one who behave thus.
∴ I am not a fool.

Invalid. *Illicit major.* The major term (*fool*) is distributed in the conclusion without being distributed in the major premise.

5. *Don't you play with fire; you will burn your fingers.*

Reduced to the logical form, the argument stands somewhat thus:

If you play with fire, you burn your fingers.
You play with fire.
∴ You burn your fingers.

Valid hypothetical-categorical syllogism that, by affirming the antecedent, affirms the consequent.

6. *The cat must be away, for the mice are about.*

Reduced to the logical form the argument stands somewhat as follows:

If the cat is away, the mice are about.
The mice are about.
∴ The cat is away.

Invalid. It involves *the fallacy of affirming the consequent.*

7. *King James I of England was a man of vast erudition. He judged everything from the formal point of view, and was always at tussle with his Parliament. His logical niceties earned for him the title of 'the wisest fool in Christendom' from the King of France. On the question of abolishing the bishopric he used to say, 'No bishops, no kings.'*

Reduced to the logical form, the King's argument stands somewhat as follows:

If there are no bishops, there are no kings.
There are no bishops.
∴ There are no kings.

Formally valid, as the consequent is affirmed by affirming the antecedent. But the Puritans always had doubts about the major premise.

8. *Seeing is believing; so I refuse to believe in God.*

Logically put, the argument would stand thus:

All things seen are believed.
God is not a thing that is seen.
∴ God is not believed.

Invalid. *Illicit major.* The major term, *believed,* is distributed in the conclusion without being distributed in the major premise.

9. *Only material bodies gravitate, and light does not gravitate.*

Logically put. it would be:

All things that gravitate are material bodies.
No light is a thing that gravitates.
∴ No light is a material body.

Invalid. *Illicit major.* The major term, *material body,* is distributed in the conclusion without being distributed in the major premise.

10. *Some of Galileo's contemporaries argued: There can be no truth in Galileo's assertion that Jupiter has moons, because they are invisible to men and can, therefore, have no bearing on the interests of mankind, and there can be nothing in this world that has not some relation to man.*

This is a train of reasoning that proceeds by leaps, as it were, in that it involves certain suppressed propositions. So fully stated, it would stand more or less as follows:

I. Things that are invisible are things that have no bearing on the interests of mankind.
The moons of Jupiter are things that are invisible.
∴ The moons of Jupiter are things that have no bearing on the interests of mankind.

This is formally valid, being in Barbara.

II. Nothing that has no relation to man is a thing that exists.
The moons of Jupiter are things that have no relation to man.
∴ No moons of Jupiter are things that exist.

This, too, is formally valid, being in Celarent.

One might, however, call the premises into question.

11. *You have added insult to injury, which no decent man ever does.*

An enthymeme that might be logically stated thus:

No decent man is a person to add insult to injury.
You are a person to add insult to injury.
∴ You are not a decent man.

Valid, being in Cesare.

12. *Mercy but murders, pardoning those that kill.*

This is an enthymeme. It may be shown to be in order in one way. In another way, it may be proved to be fallacious, as when it is put into the logical form thus:

Pardoning those that kill is murdering.
Pardoning those that kill is having mercy.
∴ Having mercy is murdering.

Invalid. *Illicit minor.* The minor term, *having mercy,* is undistributed in the minor premise, as it is there predicate of an A proposition. But as subject in the conclusion, which is also an A proposition, it has been distributed.

But whether upheld as formally valid, or rejected as formally fallacious, the fact remains that the major premise, in both the cases (see also p...), is open to question.

Just as dilemmas are more often fallacious than not, even so arguments in which one thing or another is suppressed are generally easy to interpret in quite different ways, and as such are scarcely useful in Logic, or in practical life.

13. *Protective laws should be abolished; for they are injurious, if they produce scarcity, and they are useless, if they do not.*

This is a dilemma, which when fully stated, might stand as follows:

If protective laws produce scarcity, they are injurious; and if they do not produce scarcity, they are useless.

Either they produce scarcity, or they don't.

∴ They are either injurious, or useless.

Formally valid, but not materially. The second consequent does not necessarily follow from the second antecedent. As a matter of fact, protective laws are never meant for actual scarcity. Thus the dilemma might be *taken by the horns,* and proved to be incorrect.

14. *If emigrants are useless, they are a burden to the colonies; and if they are useful, they are a loss to the mother country. But they are either useless, or useful. ∴ Emigration is either a burden to the colonies, or a loss to the mother country.*

This is a complex constructive dilemma. It can be rebutted as follows:

If emigrants are useless, they are not a loss to the mother country; and if they are useful, they are not a burden to the colonies.

They are either useless, or useful.

∴ Emigration is either not a burden to the colonies, or not a loss to the mother country.

15. Two and two are two numbers.
Four is two and two.
∴ Four is two numbers.

Invalid. *Fallacy of Composition.* In the major premise the term, *two and two*, has been used distributively, but in the minor premise it has been used collectively.

**N.B.** Thus the middle term, *two and two,* may be said to bear two meanings in the syllogism. And that amounts to a commission of the fallacy of ambiguous middle, which is, after all, a variety of the fallacy of four terms, which, again, is a case of the fallacy of equivocation.

16. Four is one number.
Two and two are four.
∴ Two and two are one number.

Invalid. *Fallacy of Division.* In the minor premise, *two and two*, is used collectively; but in the conclusion, it is used distributively.

**N.B.** This, too, is a case of ambiguity, or equivocation. But it is a case of what might be called 'ambiguous minor', as the minor term, *two and two,* here bears two meanings. The argument, however, is a reverse case of the fallacy of composition.

17. *A man was committed to the sessions on a charge of murder. All the prosecution witnesses broke down under fire of cross examination of the defence counsel, and declared that they had not seen him murder. They had only seen him take aim and pull the trigger of a revolver he held in*

*his hand, and the victim drop down in a pool of blood as soon as the explosion was heard. They also admitted that merely to take aim and fire a shot was not to murder a man, that if a man dropped down dead with a revolver shot in the chest, it did not necessarily mean that he was murdered, for he might as well commit suicide, or be the victim of a random shot from somewhere else. So they could not declare on solemn oath that the man was murdered, and by the defendant. They also admitted that merely to take aim and fire a shot was rather a trivial matter, for men often did so in sport, and for a man to drop down dead with a revolver shot in the chest was rather an accident, and that an accident might as well be a trivial affair, as so many daily accidents of omission and commission usually are. Thereupon the counsel pleaded as follows:*

*"My client is charged with murder. The evidence against him consists of a number of circumstances so trivial that, if you examine each separately, as you must so as to evaluate each in its proper light and not to mix up things, you must reject it as furnishing no conclusive evidence of guilt. I call upon you, therefore, to acquit the prisoner.*

This is a case of *the fallacy of composition,* as each circumstance by itself may be trivial, and may not prove the guilt; but all the circumstances taken together has quite a different story to tell.

18. *To help a man in distress is right; but to rescue a prisoner from lawful custody is to help a man in distress; therefore to rescue a prisoner is right.*

Invalid. An instance of *the fallacy of accident,* as it involves reasoning from a general case to a special one.

**N.B.**—Such fallacies also have some semblance to equivocation. In this particular instance, 'a man in distress' bears different shades of meaning; in one case it is understood to mean an

innocent person, while in the other it means one who is alleged to be guilty.

19. *Is a man infallible? No. Then every senator is liable to commit mistakes. Yes, ergo, the judgment of the Senate in this important matter is unreliable.*

Invalid. *Fallacy of composition.* The term, 'every senator,' is used distributively, but the other term, *viz.*, 'the senate', is used collectively. Here we pass from the distributive 'every senator' to the collective 'senate' without warrant.

20. *The surgeon is doing wrong in performing the operation, as that means inflicting pain on another person. and we know that it's wrong to inflct pain on another.*

Invalid. *Fallacy of Accident,* as it involves reasoning from one special case to another. Inflicting pain with a malicious intent is quite different from doing so with a good motive.

21. I am a man.

You are not what I am.

∴ You are not a man.

Invalid. It involves *the fallacy of accident,* as it is a case of reasoning from one special case to another.

22. *There can be no fire here, for there is no smoke; and wherever there is smoke, there is fire.*

Logically it would be:
If there is smoke, there is fire.
There is no smoke.
∴ There is no fire.

Invalid. It involves *the fallacy of denying the antecedent.*

23. *Beggars cannot ride, for wishes are not horses.*

Logically it would be:
If wishes were horses, beggars would ride.
Wishes are not horses.
∴ Beggars cannot ride.

Invalid, involving *the fallacy of denying the antecedent.*

24. *King Henry VII's minister, Empson, was bent on extorting money for his royal master by means fair or foul. He taxed the rich and poor alike, and employed the following dilemma, known as 'Empson's fork,' in vindication of his fell motive.*

*If the person taxed lives in a humble style, his savings must have made him rich; and if he maintains a large household, his expenditure proves him to be wealthy.*

*But either he lives in a humble style, or he maintains a large household.*

∴ *He is rich enough to pay.*

Formally valid, but materially false. A person who lives in a humble style, may do so because he is poor. So the dilemma can be *taken by the horns.*

25. *Logic is either a science or an art; but it is art; therefore it cannot be a science.*

Invalid. In a disjunctive-categorical syllogism, one of the alternatives cannot be denied in the conclusion, simply by affirming the other alternative in the minor premise, unless the alternatives are mutually exclusive.

But on this point logicians are not always agreed.

## Exercises.

1. Define fallacy. Is every mistake a fallacy?

2. Should the discussion of fallacies form a part of the discussion of logical problems? Fully discuss this question.

3. Classify fallacies.

4. Illustrate the following fallacies:—Vague conception, Inconsistent terms, Inconsistent propositions, Fallacy of definition, of division, of classification, and of many questions.

5. When are the fallacies of opposition committed? Give two examples of such fallacies. Give an example of each of the following:—

(*a*) Fallacy of conversion.
(*b*) Fallacy of obversion.
(*c*) Fallacy of complex conception.
(*d*) Fallacy of added determinants.
When do we commit fallacies incident to modal consequence?

6. Illustrate the following fallacies:—

(*a*) Fallacy of four terms.
(*b*) Fallacy of the illicit major.
(*c*) Fallacy of the illicit minor.

7. When do we commit fallacies in connection with mixed hypothetical and disjunctive syllogisms? Illustrate the fallacies of affirming the consequent and denying the antecedent.

8. Explain the following fallacies, and give two examples of each:—

(*a*) Equivocation.
(*b*) Figure of speech.
(*c*) Accident.
(*d*) Secundum quid.
(*e*) Composition.
(*f*) Division.
(*g*) Amphiboly, and Accent.
Why are they called semi-logical fallacies?

9. State each of the following reasonings in its full logical form, and test it fully:—

(*a*) The frequent blunders in assessment that are taking place prove that valuation is not an exact science.

(*b*) If virtue were knowledge, it would be capable of being taught, but where are the teachers of virtue?

(*c*) Movements of atoms are the only changes in the world we know anything about. Therefore all the changes in the world are movements of atoms.

(*d*) He is, to be sure, a highly educated man, but all highly educated men are not qualified to be inspectors.

10. Express the following arguments in their simplest logical forms, stating each in mood and figure, and point out fallacies in them, if any:—

(*a*) That colouring substance cannot be blood; it is soluble in benzol.

(*b*) He has broken his word, which no honest man ever does.

(*c*) All grasses have parallel-veined leaves, and so has the bamboo.

(*d*) That is a bee: do not touch it: it will sting you.

(*e*) The atmosphere after a thunderstorm must abound in ozone, because ozone is produced by the passage of electric sparks through the atmosphere.

11. State any three of the following arguments in their full logical form, showing figure and mood, and examine their validity:—

(*a*) *The Critias* cannot be a work of Plato, it is of small literary value.

(*b*) The last speaker is opposed to the motion, but every sensible man wishes it passed.

(*c*) I shall be admitted, because I have passed in the first division and only first division candidates will be admitted.

(*d*) The more abundant a thing becomes the more will it sink in value: and silver is becoming more abundant every day.

(*e*) I shall not pass this examination, for although I should have done so if I had read Mill's Logic, I have not read the book.

12. State the following arguments in their logical form, and test them:—

(*a*) How can anyone maintain that pain is always an evil, who admits that remorse involves pain, and yet may sometimes be a real good?

(*b*) Where there is no law, there is no injustice.

(*c*) The farther your neighbour lives from you, the more you are bound to be true in your dealings with him, because your power over him is greater in proportion to his distance.

(*d*) You cannot define the sun, for a definition must be clearer than the thing defined, and nothing can be clearer than the source of all light.

(*e*) Gibbon was not very talented, for a successful author must be either very industrious or very talented, and Gibbon was very industrious.

13. Test the following:—

(*a*) India comprehends Bengal, Bengal does not comprehend Bombay, India therefore does not comprehend Bombay.

(*b*) Theft cannot be a crime, for it was encouraged by the laws of Lycurgus.

(*c*) The child of Themistocles governed her mother, she governed her husband, he Athens, and Athens Greece; the child of Themistocles therefore governed Greece.

(*d*) None but the express train stops at this station, and as the last train did not stop, it cannot have been the express train.

(*e*) If all men were honest, laws would be unnecessary, but since laws are necessary it follows that no men are honest.

(*f*) Protective laws should be abolished, for they are injurious if they produce scarcity, and they are useless if they do not.

14. Test the following:—

(*a*) The revenues of Vitellius were spent on the necessaries of life; for they were spent on meat and drink, and everyone must admit that meat and drink are the necessaries of life.

(*b*) Haste makes waste, and waste makes want; therefore a man never loses by delay.

(*c*) Since the virtuous alone are happy, he must be virtuous if he is happy; and he must be happy if he is virtuous

(*d*) To allow every man unbounded freedom of speech is advantageous to the State, for it is highly conducive to the interest of the community that each individual should enjoy an unlimited liberty of expressing his sentiments.

(*e*) What is Protestantism? It is only loyalty to the sovereign;

for were not the Protestants loyal to Elizabeth in her struggle with Spain?

15. Test the following arguments:—

(*a*) If a man is educated he does not want to work with his hands; consequently, if education is universal, industry will cease.

(*b*) We have no right to treat heat as a substance, for it may be transformed into something which is certainly not a substance, namely, mechanical work.

(*c*) Learned men sometimes become mad: but, as he is not learned, there is no danger of his insanity.

(*d*) Men who live in society are liable to go wrong morally, therefore to be virtuous one must renounce society.

16. Test the following arguments:—

(*a*) It is strange that in a country like India, where millions of men die every year of plague, malaria and other diseases, people are loath to join the army, seeing that casualties of warfare cannot possibly be higher.

(*b*) What John Smith advocates must be a wise course, since he is a senator and the Senate is undoubtedly a wise body.

(*c*) Sankaracharyya was a great religious reformer and teacher, and as he was a bachelor, we may safely conclude that anyone leading a single life may become a great reformer and teacher.

(*d*) The whole family has been vaccinated, yet some have had smallpox; it is clear, therefore, that vaccination is no safeguard.

(*e*) Improbable events happen almost every day, but what happens almost every day is a very probable event. Hence improbable events are very probable events.

17. Test the following arguments:—

(*a*) The standard of the Calcutta University must be low, since the percentage of success at its examinations is comparatively high; and it is a well-known fact that the percentage of success is high when the standard is low.

(*b*) The post-graduate classes ought to be abolished, for if graduates have a real craving for knowledge, they will continue their studies whether there be such classes or not; and if they have no such craving, they will not continue their studies in spite of such classes.

(*c*) Is man infallible? No. Then every senator is liable to make mistakes? Yes, Ergo, the judgement of the Senate in this important matter is unreliable.

(*d*) Every man has the right of private judgement, therefore every examiner is quite at liberty to examine answer papers as he likes.

(*e*) Only the best graduates of the University are nominated for executive service under Government, but Jones has not been so nominated. Therefore he is not one of the best graduates of the University.

18. Test the following:—

(*a*) Beggars who have no property cannot claim the protection of law, for all laws are made for the protection of property.

(*b*) Roman Catholicism is but another name for disloyalty to the sovereign, for were there not many Roman Catholic plots in England to depose Queen Elizabeth?

(*c*) Bolshevism does not exist in India, for if it existed, its existence would have been traced by this time.

(*d*) Perhaps fate was against Hannibal, or how could the victor of Cannae be vanquished at Zama?

19. Test the following:—

(*a*) Swaraj or self-government can be attained only by doing away with servitude. As, however, the University in training young men renders them fit for public service, the first step towards the attainment of Swaraj must be the destruction of the University.

(*b*) If all men were capable of perfection, some would have attained it, but none having done so, none is capable of it.

(*c*) Surely what a man has done a man can do. Was not Hercules a man? Yes. Then why shall I not be able to do what he did?

(*d*) How can you deny that John is deeply versed in the subject? You see he criticises all topics pertaining to it, and evidently such critics alone can be said to have a thorough insight into the subject?

(*e*) If a little economy would save half of our expenses, a still greater economy would certainly save all.

20. Test the following:—

(*a*) What a man has done a man can do; this child therefore must be able to cross the English Channel, since Captain Webb was able to do so.

(*b*) If students did their work well, examinations would not be necessary. But we find as a matter of fact that examinations are necessary. It is therefore evident that students never do their work well.

(*c*) Ramamurthi might well have been sent to quell the riot, since he is more than a match for the most powerful man.

21. Test the following:—

(*a*) John must be thoroughly honest, since he is very loud in denouncing evil, and only those who so denounce it are known to be honest.

(*b*) James is evidently guilty of a breach of faith in voting against the grant of the franchise to women, since as a member of the provincial legislative council he is bound to represent the views of his constituents, and they have clearly declared at a public meeting that women should have votes.

(*c*) Six and seven are evidently one number, for six and seven are thirteen, and thirteen is one number.

(*d*) Is it not a virtue to help the distressed? Certainly it is. Are not these criminals in police custody in distress? Yes. Then surely I ought to rescue them from their custody.

(*e*) The noble Brutus hath told you Caesar was ambitious.

If it were so. it was a grievous fault. But Brutus says he was ambitious, and Brutus is an honourable man.

22. Examine the following arguments, and indicate the fallacies (if any) which may lurk in them:—

(*a*) A body moves if it is propelled from behind This body is propelled from behind, since it moves.

(*b*) The killing of living creatures is sometimes necessary; murder is a killing of living creatures; therefore murder is necessary sometimes.

(*c*) Aristotle was a great logician, since he was a philosopher, and all great logicians are philosophers.

(*d*) The cat must be away, since the mice are playing about, for when the cat is away the mice will play.

(*e*) He has got fever, for his skin is hot.

(*f*) Matter does not exist, since it does not think, and whatever thinks exists.

(*g*) Being an Indian, he must be a Hindu, for only Indians are Hindus.

23. Examine the following arguments:—

(*a*) His losses must be cheering, for they are light, and light is always cheering.

(*b*) He who is most hungry eats most. He who eats least is most hungry. Therefore he who eats least eats most.

(*c*) The doctor has prescribed poison for the patient, for he has prescribed alcohol, and is not alcohol a sort of poison?

(*d*) Socrates was wise, and wise men alone are happy, there fore Socrates was happy.

(*e*) All men are angels, for they are rational beings, as all angels are.

(*f*) He is not superstitious, since all ignorant men are superstitious and he is not ignorant.

(*g*) You must have convicted the prisoner, for you were a member of the committee which convicted him.

24. Examine the following arguments, pointing out the fallacy (if any) in each case:—

(*a*) All virtuous men are happy; therefore he is virtuous, being happy.

(*b*) God created man, man created sin, therefore God created sin.

(*c*) Bats have no wings, since they are not *birds, and all birds* have wings.

(*d*) He should apply for the post, for he is a graduate and graduates alone should apply for the post.

(*e*) If one is guilty one trembles with fear; therefore the accused must be guilty, for he is trembling with fear.

25. State any of the following arguments in the logical form, and show whether they are correct or not, giving reasons:—

(*a*) The general rules the army, the general's wife rules the general, therefore the general's wife rules the army.

(*b*) All nations which are self-governed are prosperous, India is not self-governed, therefore India is not prosperous.

(*c*) Sisters of charity are liable to punishment, because they beg money from people, and beggars are punishable according to law.

(*d*) Judging from the very large number of failures, the test must have been very stiff.

(*e*) Material bodies are invisible, for they are nothing but atoms, and atoms are invisible.

26. Test the following:—

(*a*) All these men are sufficient for the job, you are one of them, and are quite sufficient for it.

(*b*) You cannot undo the knot, for whatever is done can never be undone.

(*c*) The Divine Law commands us to honour kings, Louis XIV is a king, therefore the Divine Law commands us to honour Louis XIV.

(*d*) If a man be perfectly happy he is virtuous; James is virtuous; therefore James is perfectly happy.

(*e*) The brave alone can face danger. therefore he is not brave, for he cannot face danger.

(*f*) This substance cannot be gold, for it is not malleable.

(*g*) If he pleads that he did not steal the goods, why, I ask, did he hide them, as no thief ever fails to do?

27. Test the following:—

(*a*) Few soldiers can be considered heroes, for anyone who is incapable of fear must be called a hero, and but few soldiers can be said to be incapable of fear.

(*b*) Women as a class have not been hitherto equal in intellect to men, therefore they are necessarily inferior.

(*c*) Mathematical study undoubtedly improves the reasoning powers, but as the study of logic is not a mathematical study, we may infer that it does not improve the reasoning powers.

(*d*) He that is of God heareth God's words; ye thererefore hear them not, because ye are not of God.

(*e*) To play all day is a proof of great idleness, so this violinist must be a very idle person.

(*f*) Prosperity is the reward of industry, and honour that of honesty; but James is neither prosperous nor is honoured: therefore James is neither industrious nor honest.

28. Test the following:—

(*a*) Everything is allowed by law which is morally right; indulgence in pleasure is allowed by law; therefore indulgence in pleasure is morally right.

(*b*) All the works of Shakespeare cannot be read in a day; therefore the play of Hamlet, being one of the works of Shakespeare, cannot be read in a day.

(*c*) He who calls you a man speaks truly, he who calls you a fool calls you a man, therefore he who calls you a fool speaks truly.

(*d*) If he says that he did not tell a lie, why, I ask, did he look abashed, as liars always do?

(*e*) Every hen comes out of an egg, every egg comes out of a hen, therefore every egg comes out of an egg.

29. Analyse and test the following arguments, and mention the fallacy, if any, which they involve:—

(*a*) It hoots, so it is only an owl.

(*b*) Learned men sometimes become mad, but as he is not learned, there is no danger to his sanity.

(*c*) All men are not industrious; Brown is industrious, so he cannot be a man.

(*d*) Some mineral compounds are not decomposed by heat; therefore no organic substances are mineral products, since all organic substances are decomposed by heat.

30. Test the following:—

(*a*) If you work hard, you will get a prize; therefore you must have worked hard, for you have got a prize.

(*b*) Some poisons are vegetable; no poisons are useful drugs; therefore some useful drugs are not vegetable.

(*c*) The soul always thinks, inasmuch as to think is its nature as a rational being.

(*d*) I can afford to buy these books, I can afford to buy these pictures, I can afford to buy these statuettes; the books, the pictures, and the statuettes are all that I at present wish to buy; I can, therefore, buy everything that I want to buy.

(*e*) I know he was a Bohemian, for he was a good musician and Bohemians are always good musicians.

(*f*) All men have equal rights, therefore if A has a right to ten thousand a year, so has B.

31. State the following arguments in their strictly logical form, and test their validity, mentioning the fallacies (if any) involved in them:—

(*a*) He must be a Scotsman, for no Scotsman can see the force of a joke.

(*b*) I do not derive my opinion from the newspapers, for I never read any of them.

(*c*) We know that the policy was wrong, for otherwise it would not have failed.

(*d*) If the charge is false, the author of it is either ignorant or malicious; but the charge is true; therefore he is neither.

(*e*) Warm countries alone produce wine; Spain is a warm country; therefore Spain produces wine.

(*f*) Lawyers cannot afford to be sincere, and no one who is insincere is trustworthy. Therefore no trustworthy people will be found to be lawyers.

(*g*) Mercy but murders, pardoning those that kill.